

# Partial Solution Based Constraint Solving Cache in Symbolic Execution

Zhenbang Chen

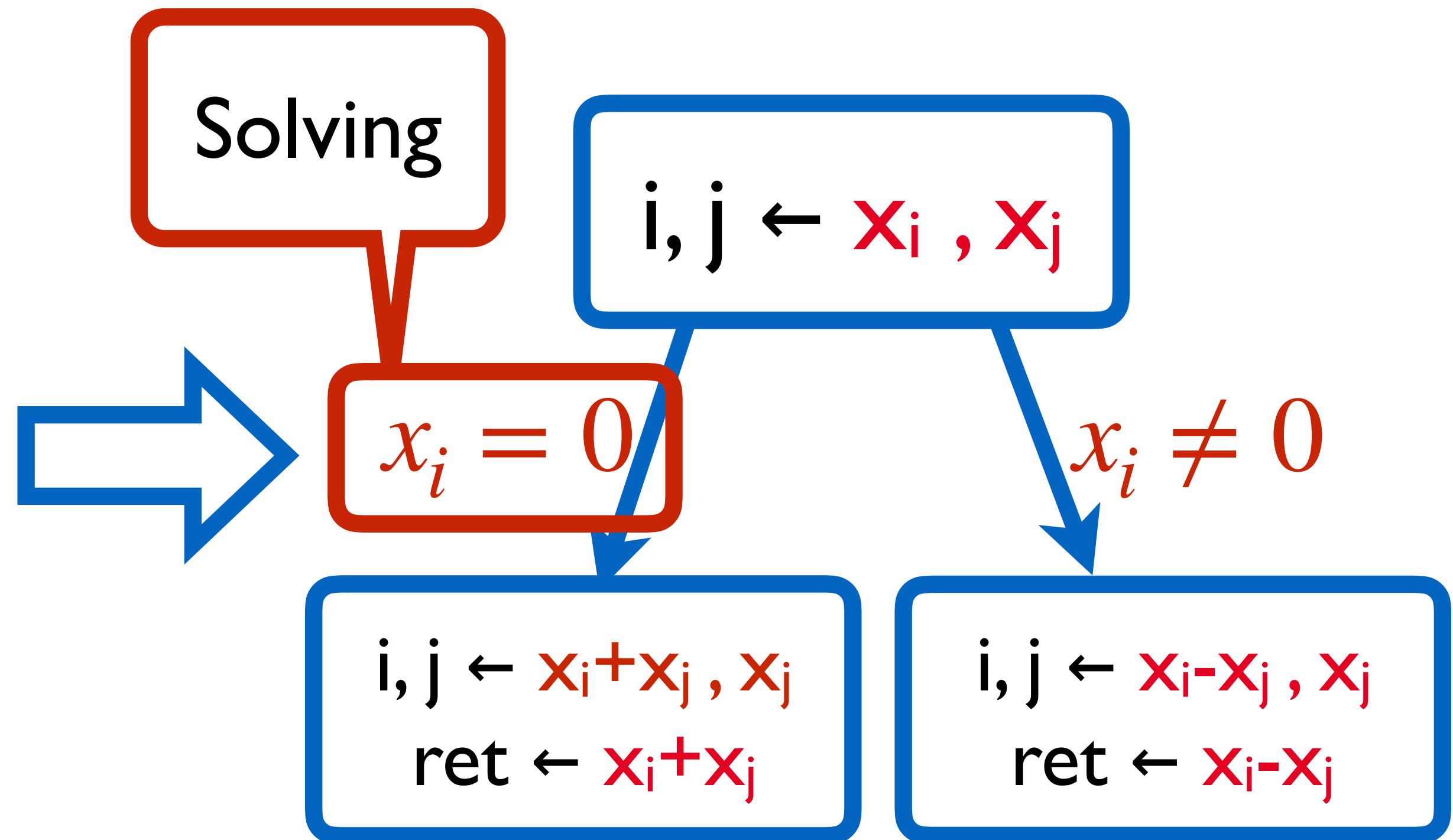
zbchen@nudt.edu.cn

*Joint work with Ziqi Shuai, Kelin Ma, Kunlin Liu, Yufeng Zhang, Jun Sun and Ji Wang*



# Symbolic Execution

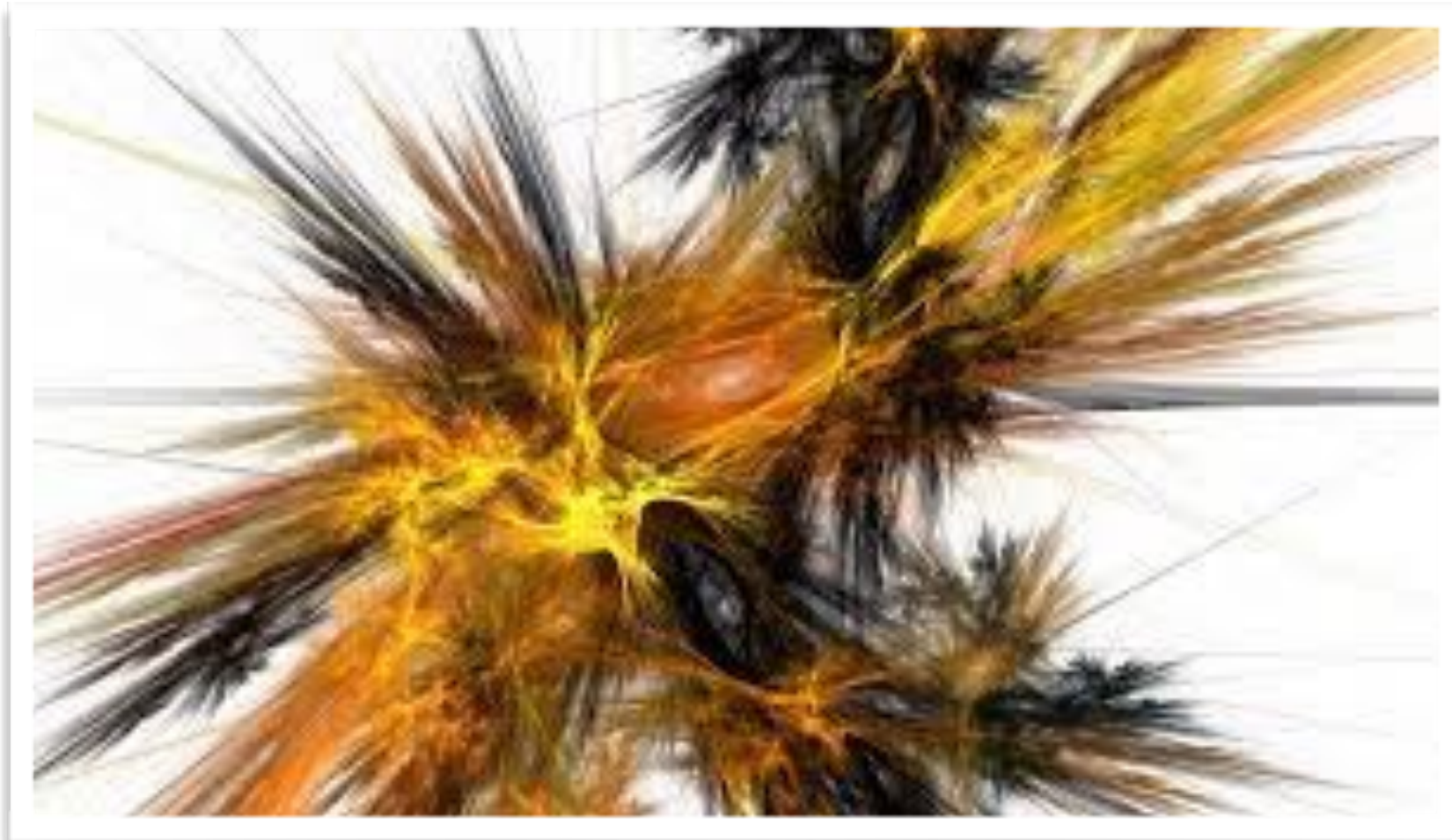
```
int foo(int i, j) {  
  if (i == 0) {  
    i = i + j  
  } else {  
    i = i - j  
  }  
  return i  
}
```



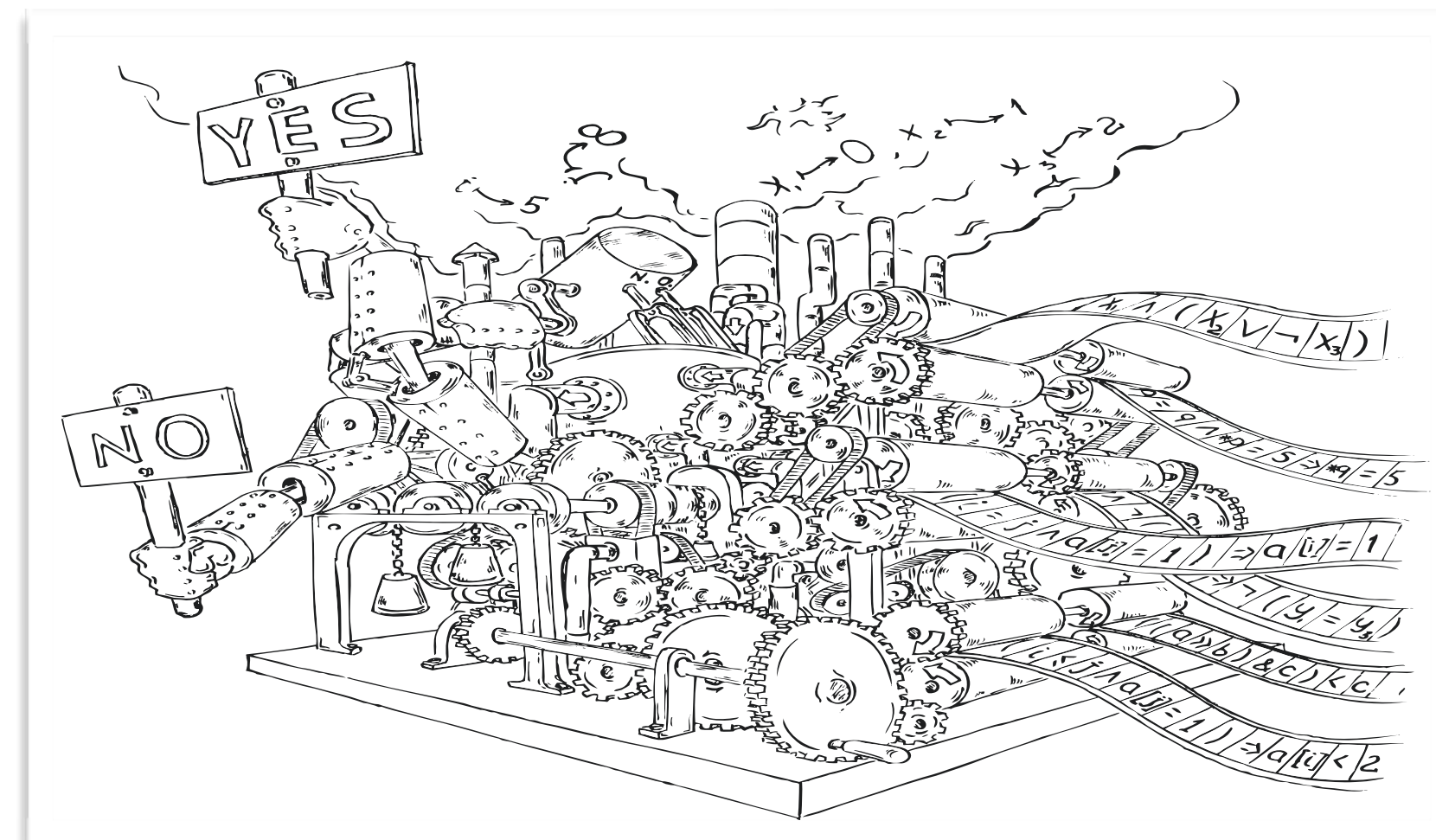
Constraint solving is the enabling technique

# Challenges of Symbolic Execution

Path explosion



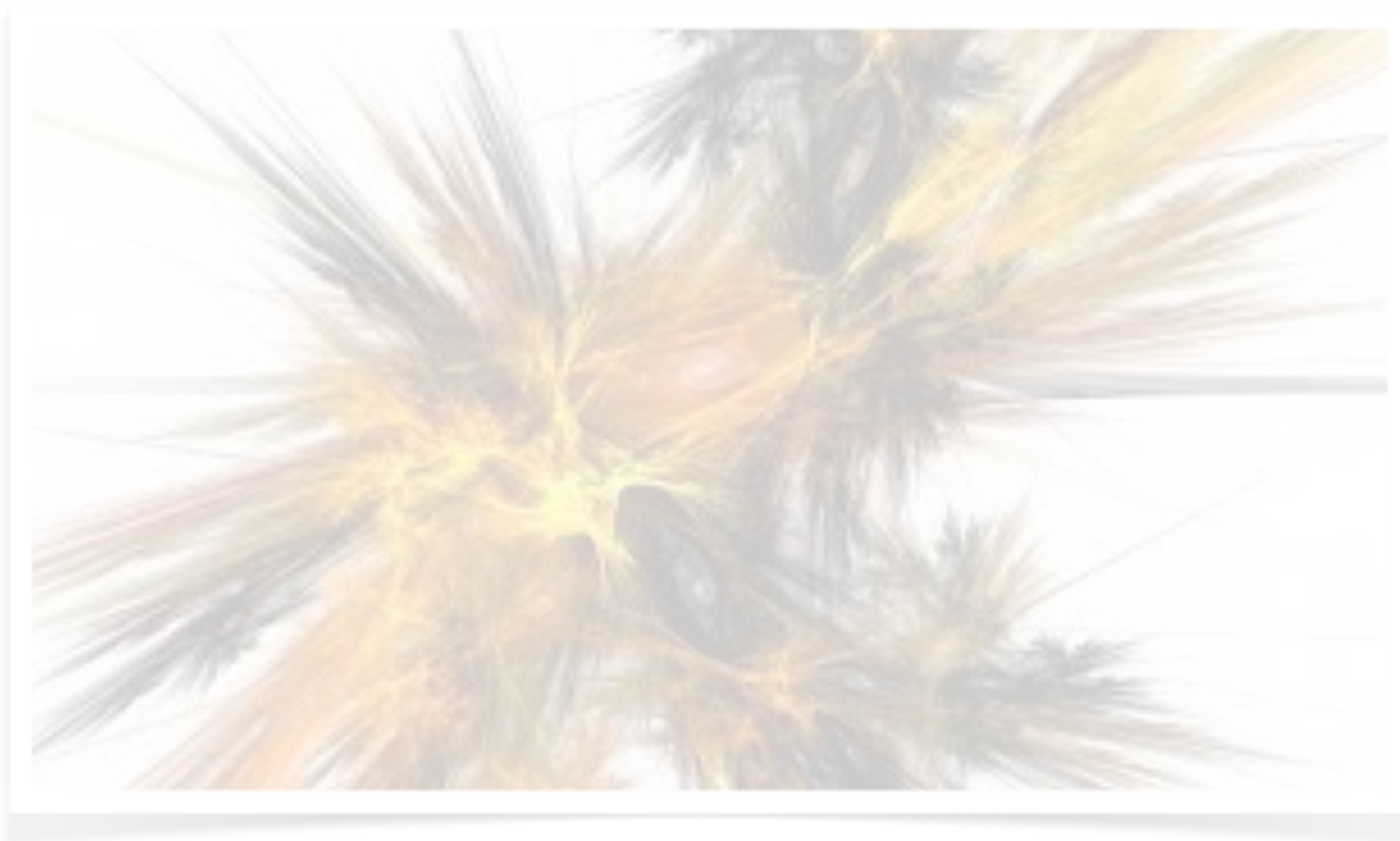
Constraint Solving



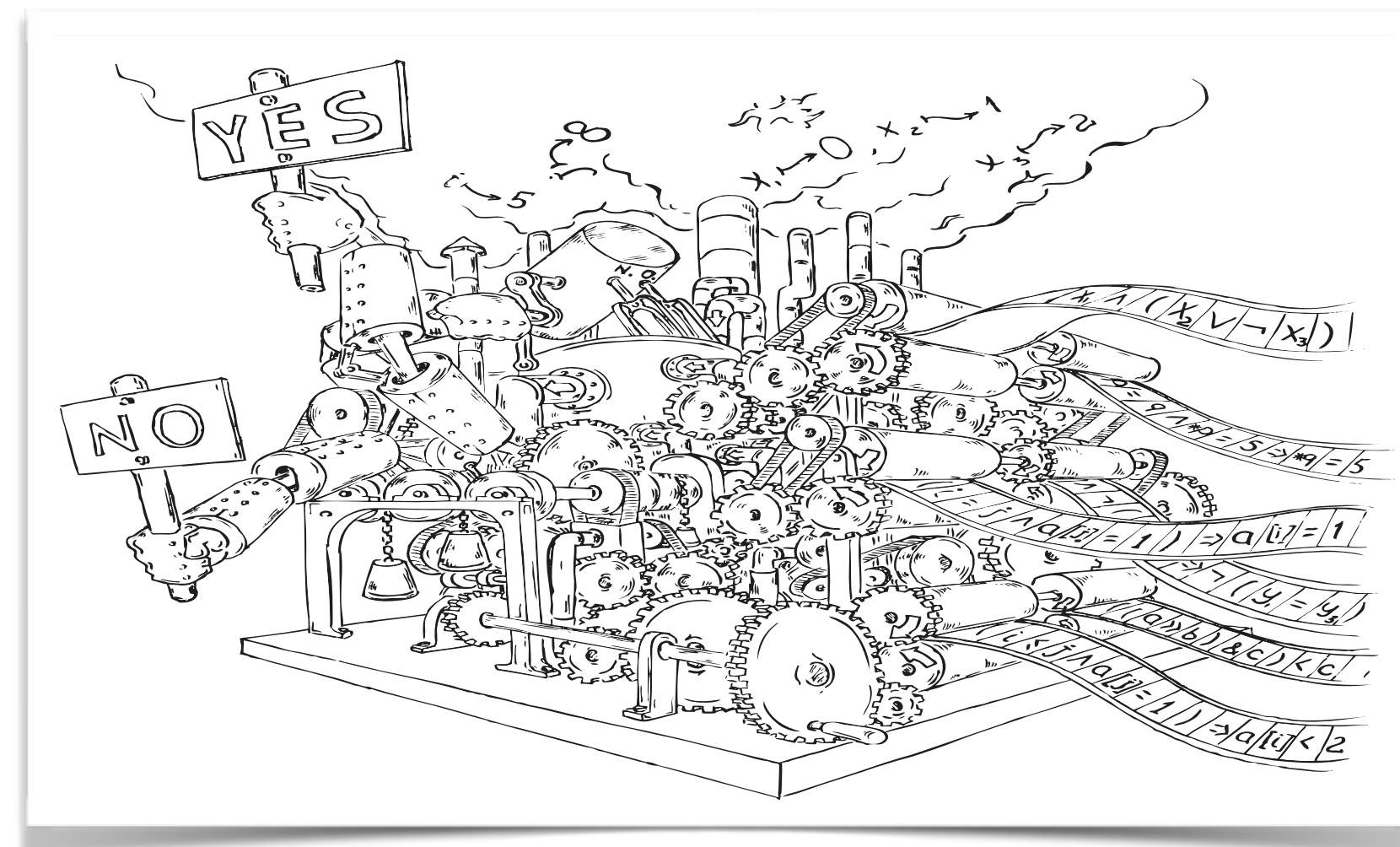
*Decision Procedures An Algorithmic Point of View, Second Edition, 2016*

# This Talk's Target

Path explosion



Constraint Solving



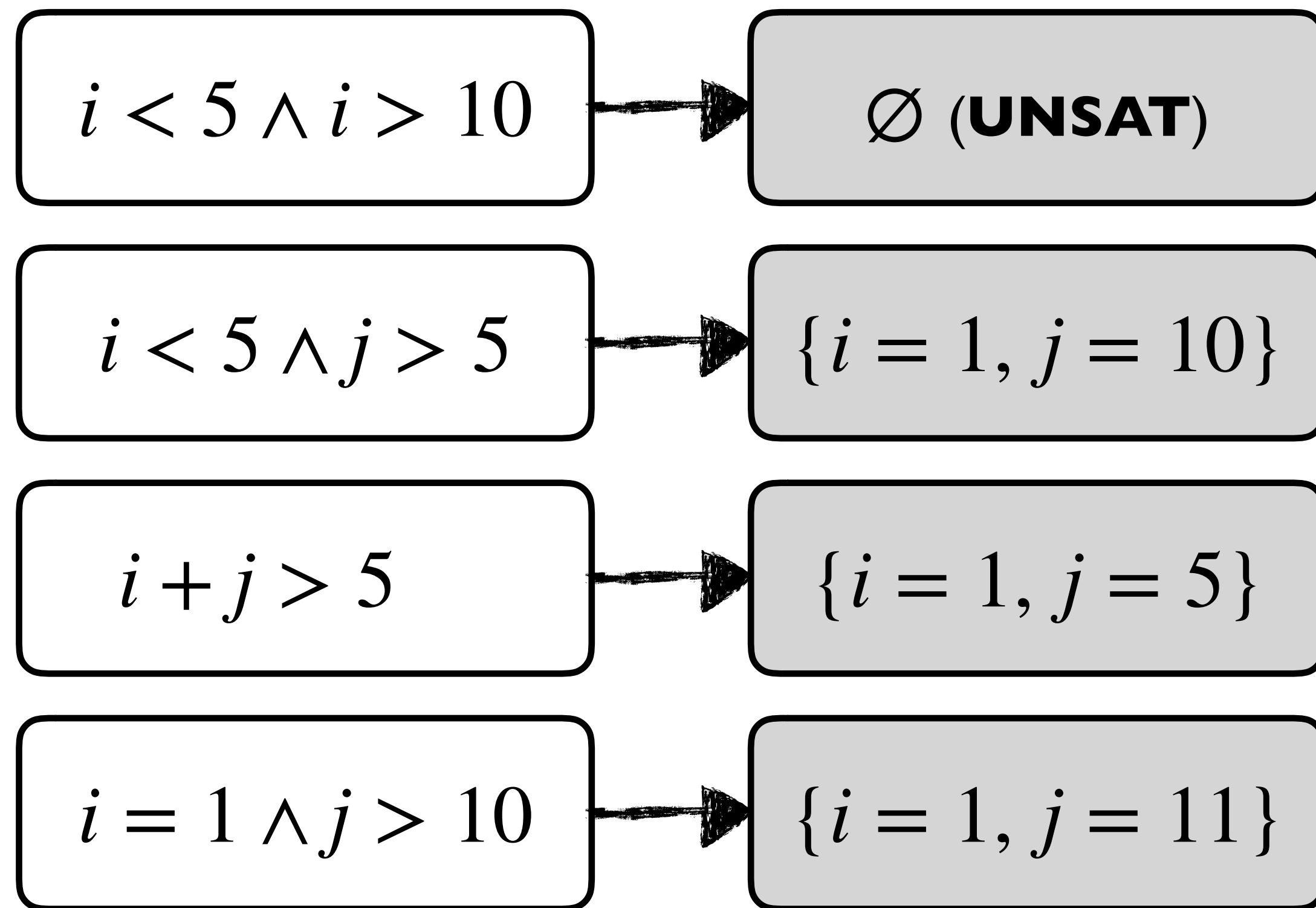
*Decision Procedures An Algorithmic Point of View, Second Edition, 2016*

# Constraint Solving Optimizations

- Optimizing SMT queries
  - Word-level simplifications [OSDI'08][ISSTA'17]
  - Concretization and abstraction [ISSTA'11][Security'15]
- Reducing SMT solver invocations
  - Speculative symbolic execution [ISSRE'12]
  - Caching mechanism [OSDI'08][FSE'12][ISSTA'15]

# Caching Mechanism

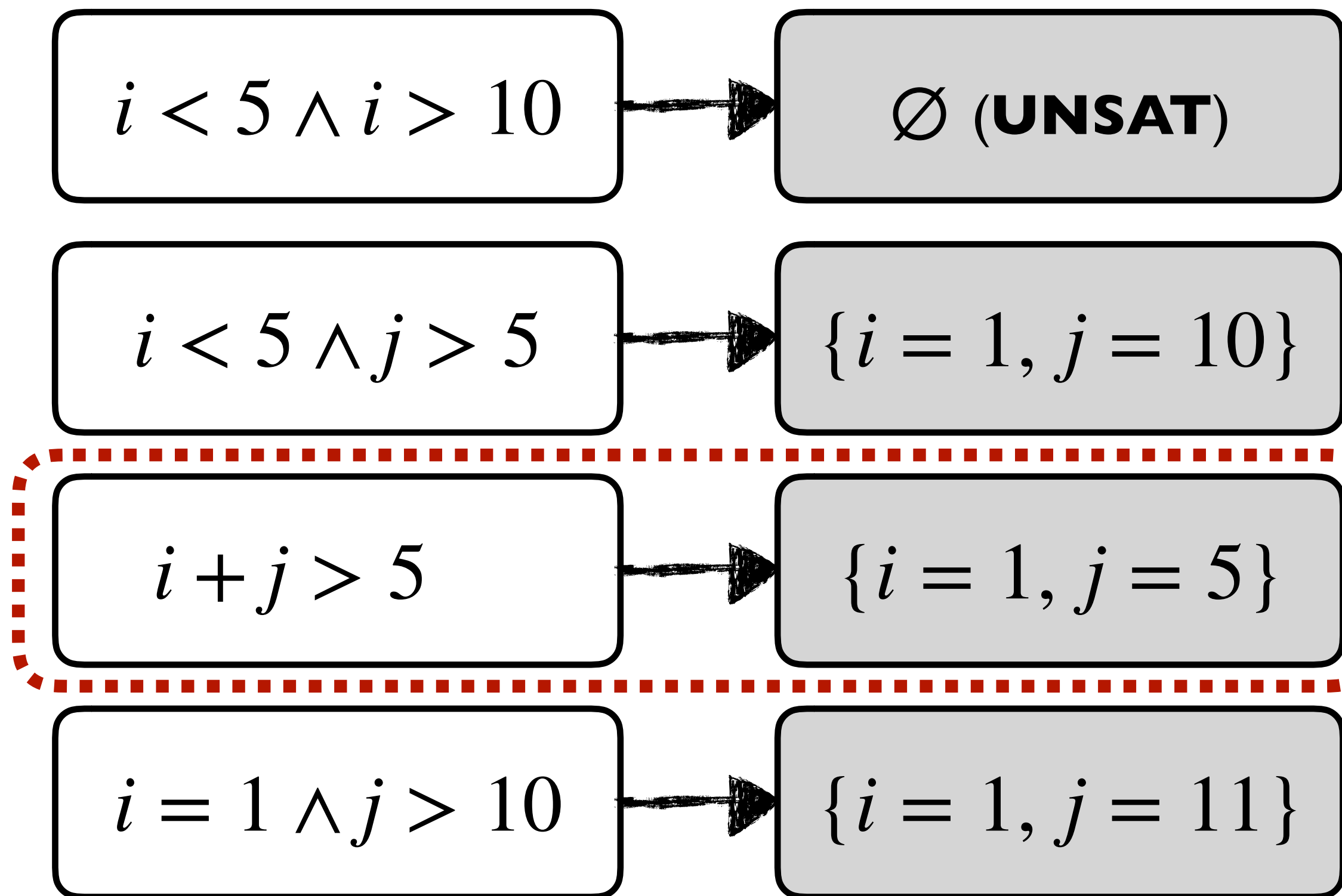
## Constraint Solving Cache



# Caching Mechanism

Constraint Solving Cache

If we encounter...

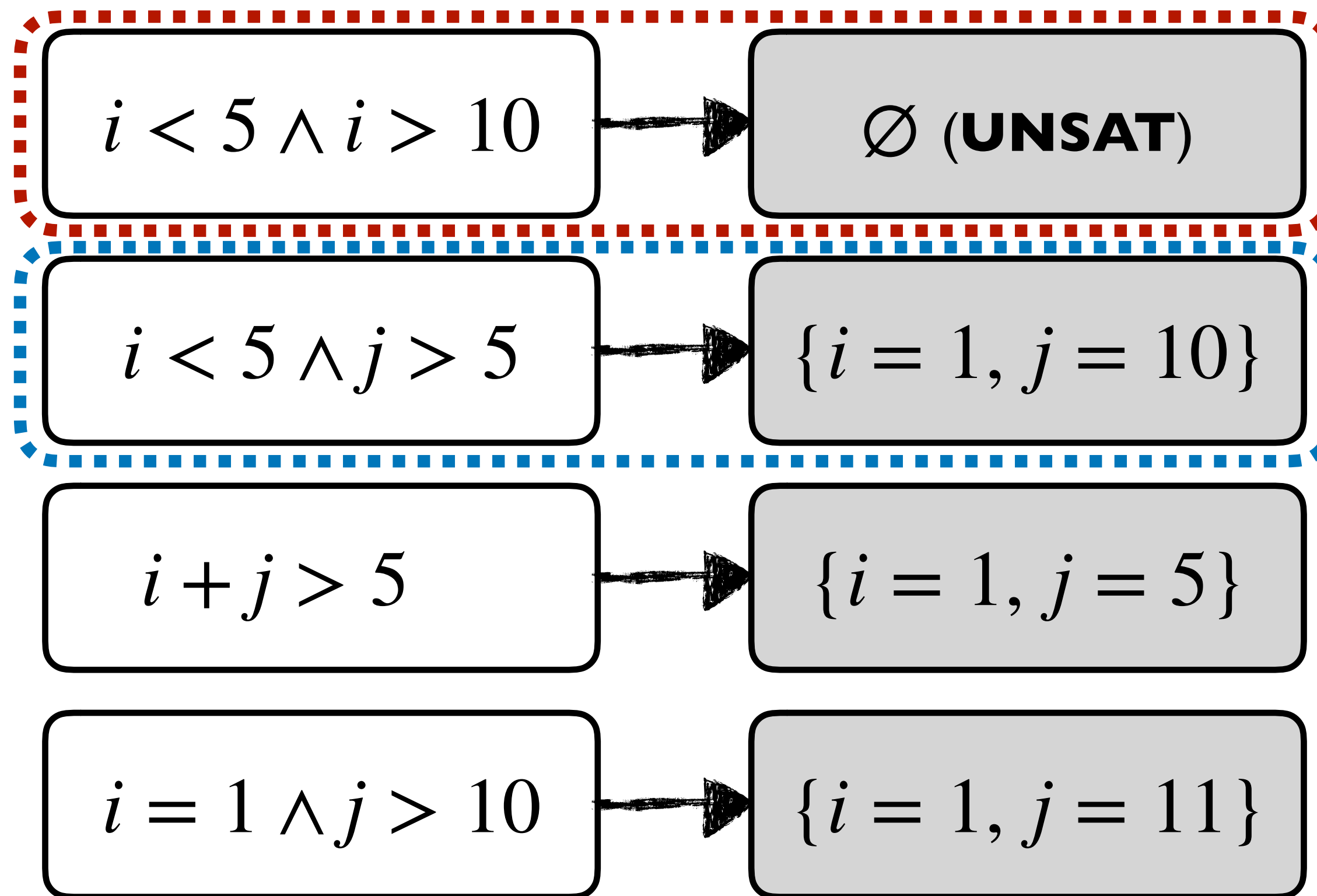


$i + j > 5$

**Strict Reusing**

# Caching Mechanism

Constraint Solving Cache



If we encounter...

$$i + j > 5$$

$$i < 5 \wedge i > 10 \\ \wedge i + j > 20$$

$$i < 5 \wedge j > 5 \\ \wedge i + j > 10$$

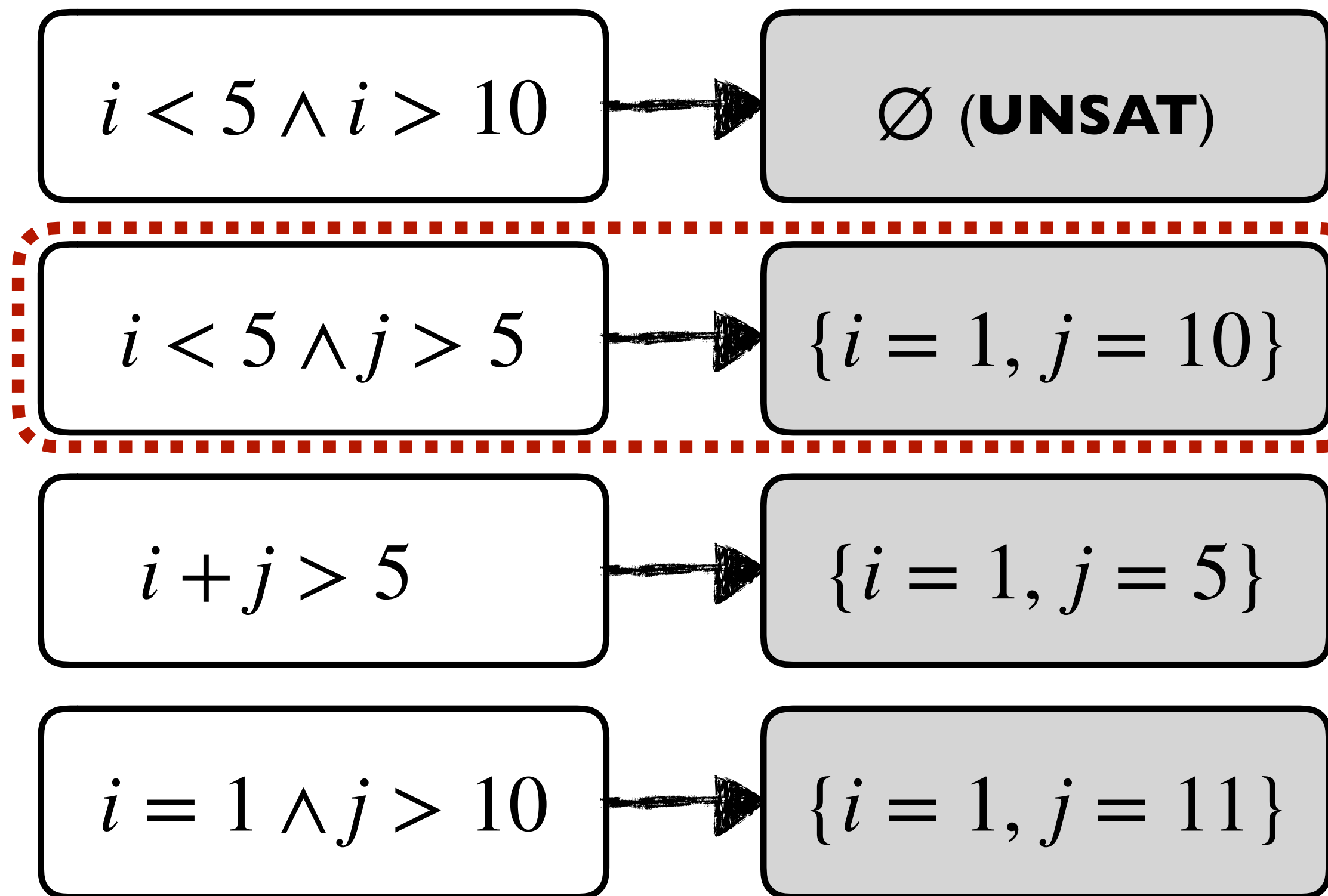
**Strict Reusing**

**Subset-based Reusing**



# Caching Mechanism

Constraint Solving Cache



If we encounter...

$$i + j > 5$$

$$i < 5 \wedge i > 10 \\ \wedge i + j > 20$$

$$i < 5 \wedge j > 5 \\ \wedge i + j > 10$$

$$i < 5$$

**Strict Reusing**

**Subset-based Reusing**

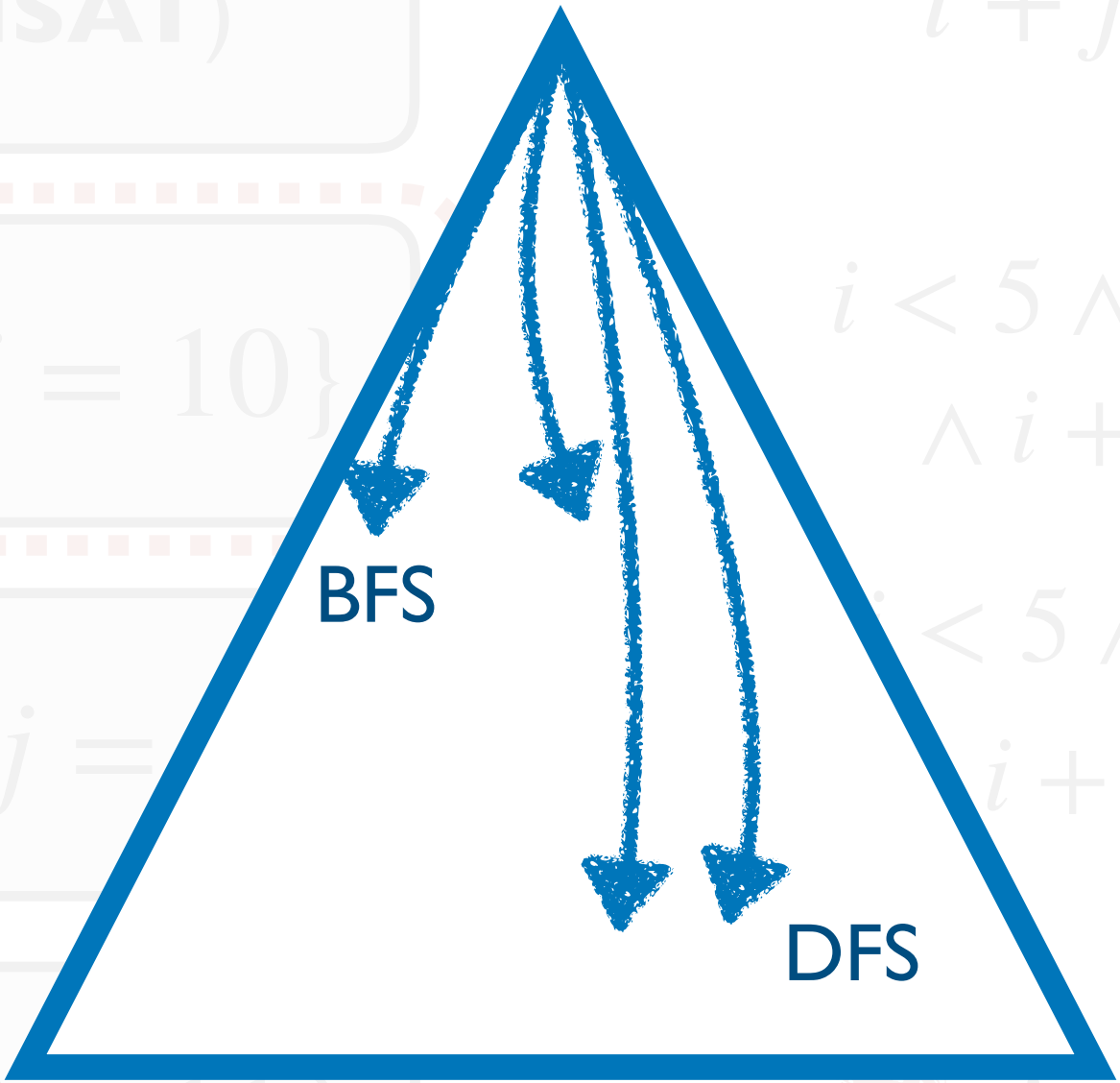
**Superset-based Reusing**

# Problem

The effectiveness of the cache depends on many factors



Program Structure



Search Heuristic

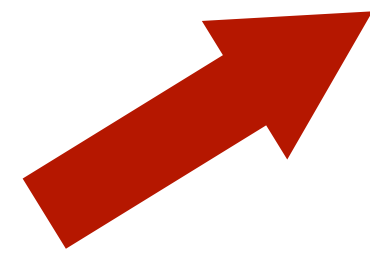


Constraint Solver

# Related Works

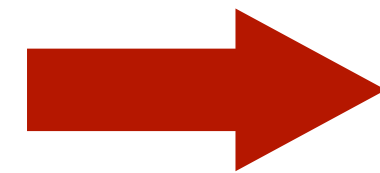


constraint  
solving cache



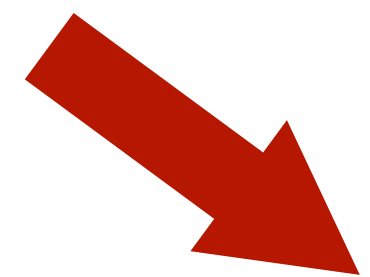
Reusing across runs

Green[FSE'12], GreenTrie[ISSTA'15]



Reusing through  
imprecise matching

Utopia[TSE'21]



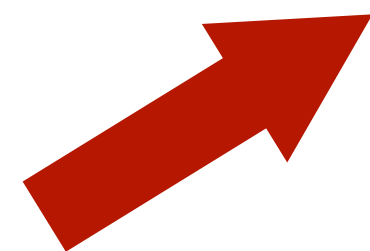
Handling address-  
dependent queries

Trabish et. al[ICST'21]

# Related Works

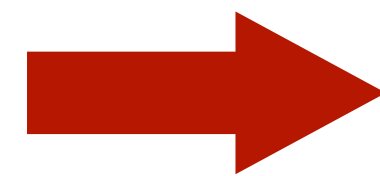


constraint  
solving cache



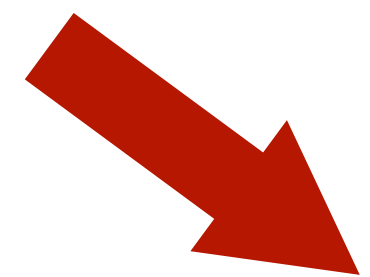
Reusing across runs

Green[FSE'12], GreenTrie[ISSTA'15]



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Handling address-  
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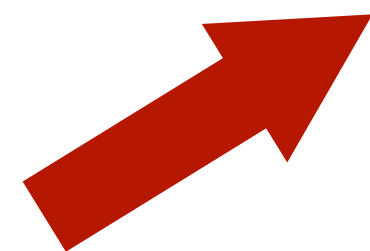
Improve the  
usability of  
existing  
solutions

Not general

# Related Works

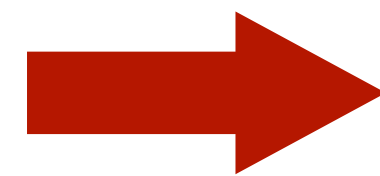
**CACHE**

constraint  
solving cache



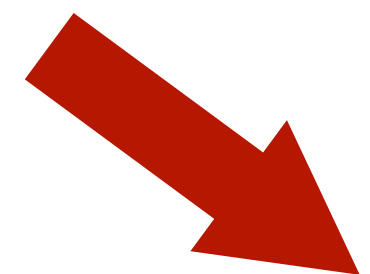
Reusing across runs

Green[FSE'12], GreenTrie[ISSTA'15]



Reusing through  
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Utopia[TSE'21]



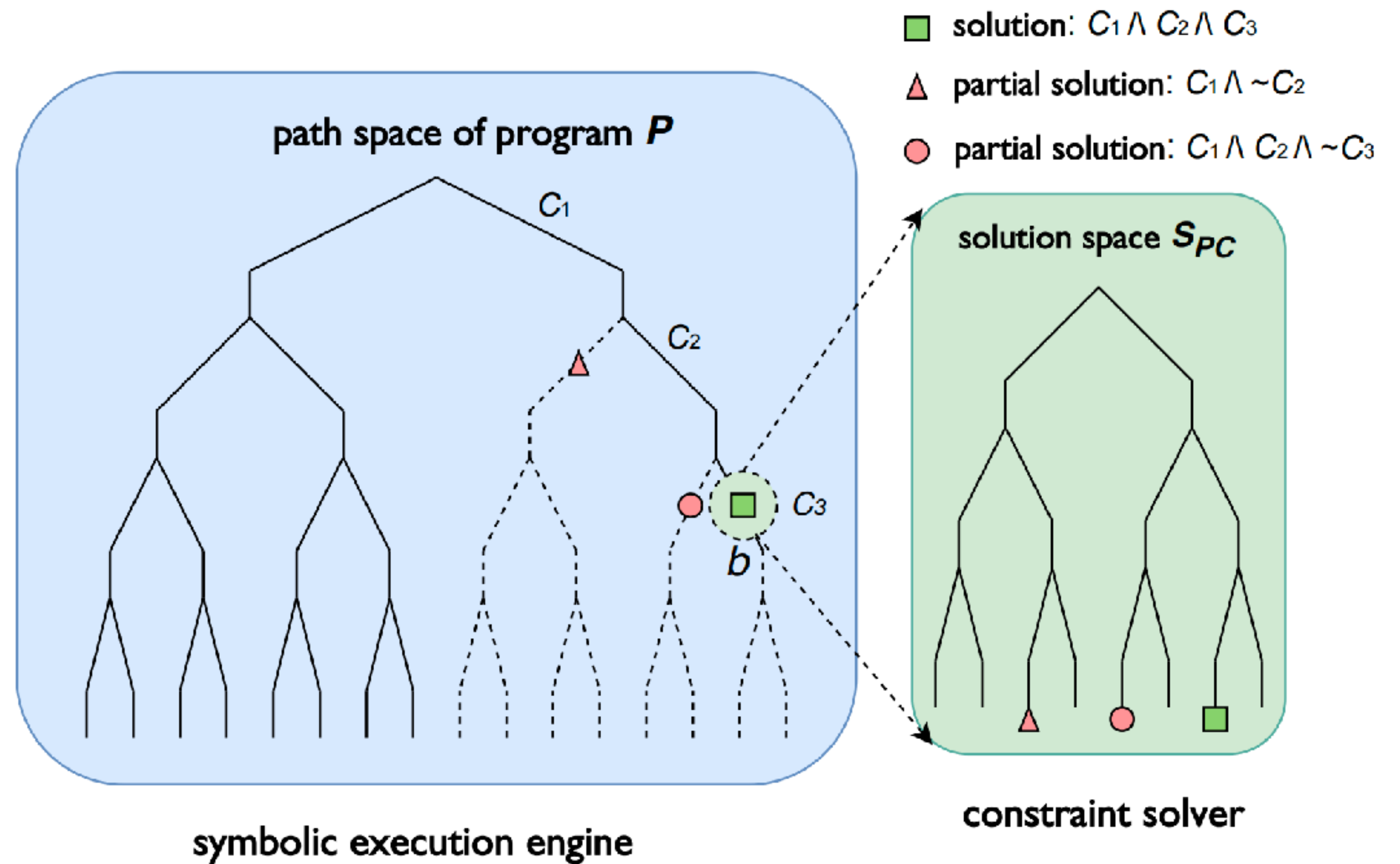
Handling address-  
dependent queries

Trabish et. al[ICST'21]

Generally  
improving  
caching's  
effectiveness  
further is still a  
challenging  
problem

# Our Key Insight (1/2)

- **Partial Solutions<sup>[1]</sup> (PS)**
- Intermediate values in constraint solving
- Satisfy some sub-constraints



[1] Multiplex Symbolic Execution: Exploring Multiple Paths by Solving Once, ASE'20

# Our Key Insight (2/2)

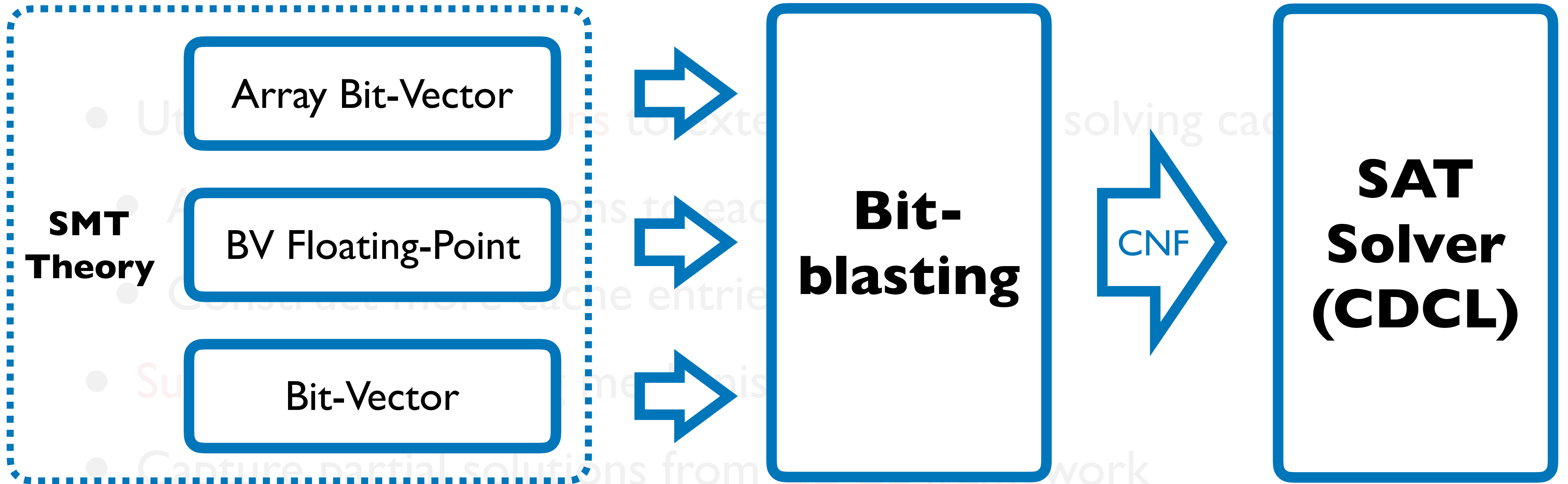
- Constraint solving often produces lots of **partial solutions**
  - Abstraction refinement-based array theory solving
  - Optimization-based floating-point solving
  - Simplex-based QF\_LIA theory solving
  - Conflict-driven clause learning (CDCL) algorithm
  - ...

# Key Idea

- Utilize **partial solutions** to expand constraint solving cache
  - Attach **more solutions** to each cache entry
  - Construct **more cache entries**
- **Subset-based reusing** mechanism
- Get partial solutions from **CDCL** framework

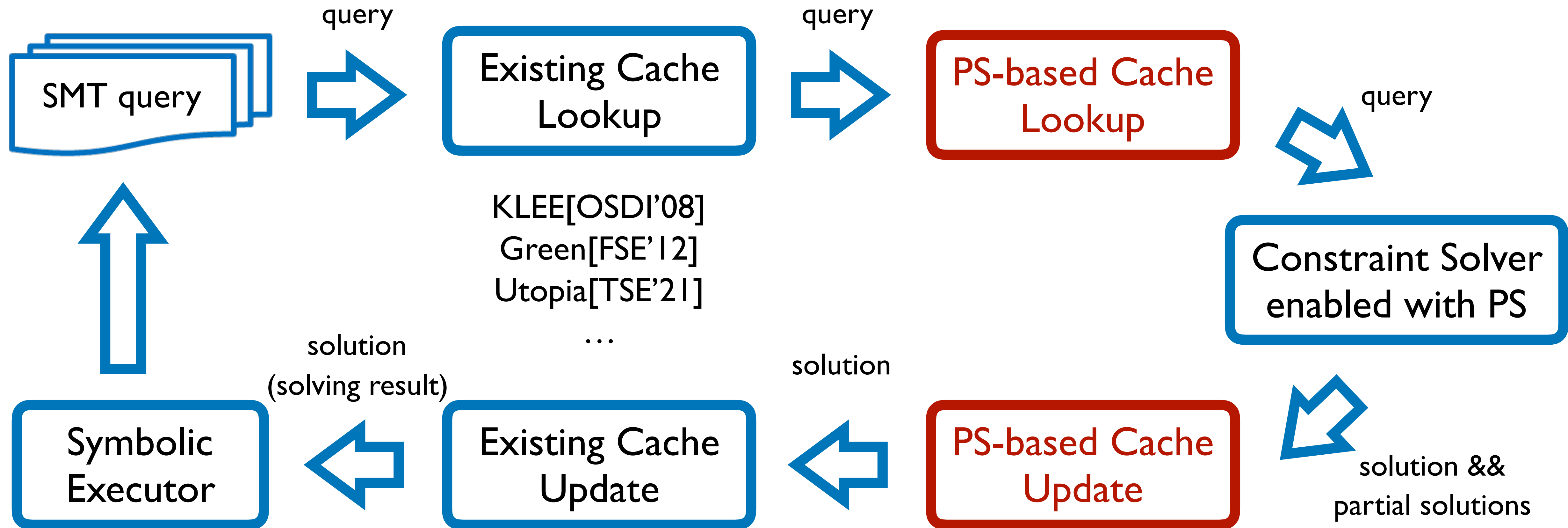


# Key Idea

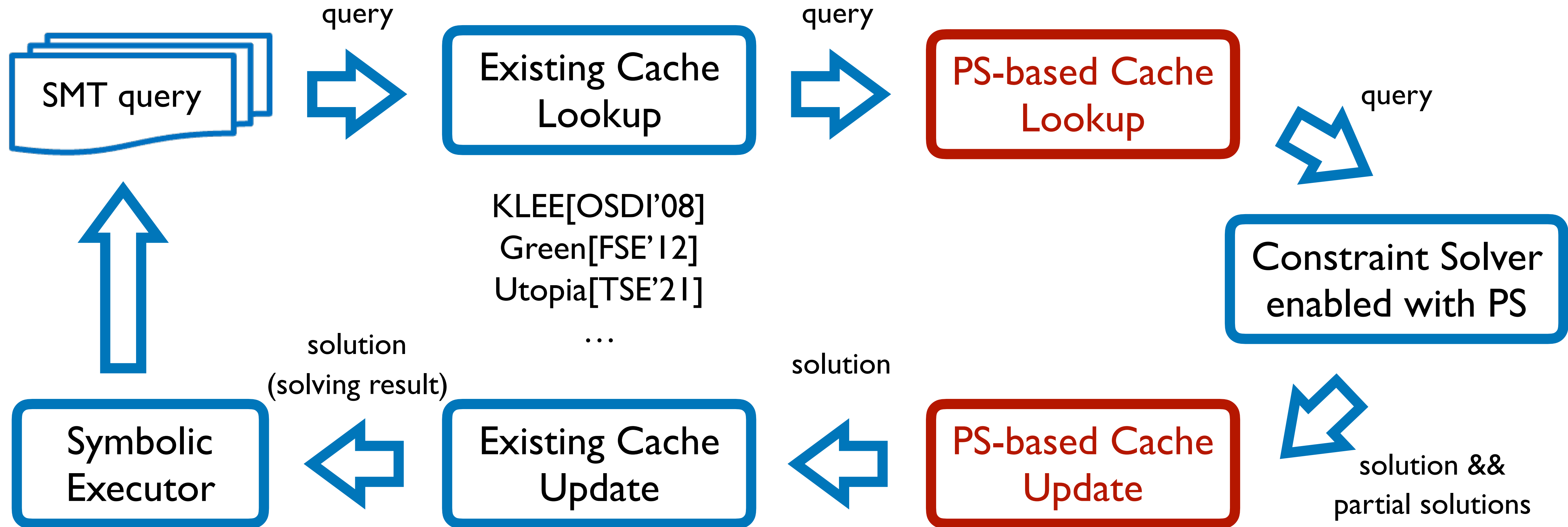


Dominant approaches to SMT rely on calling a CDCL-based SAT solver

# Partial Solution-Based Caching



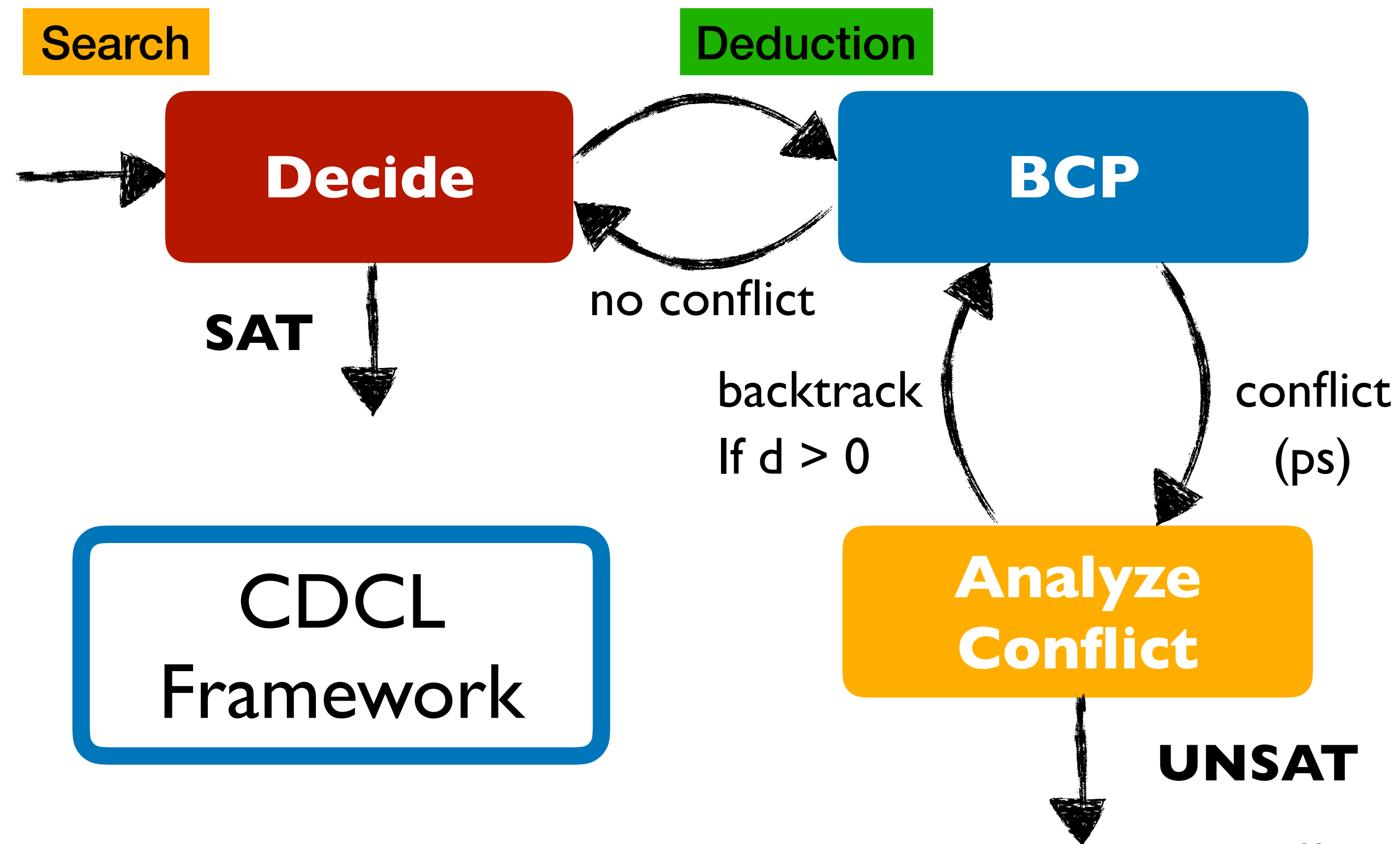
# Partial Solution-Based Caching



**PS-based caching is orthogonal to existing caching techniques**

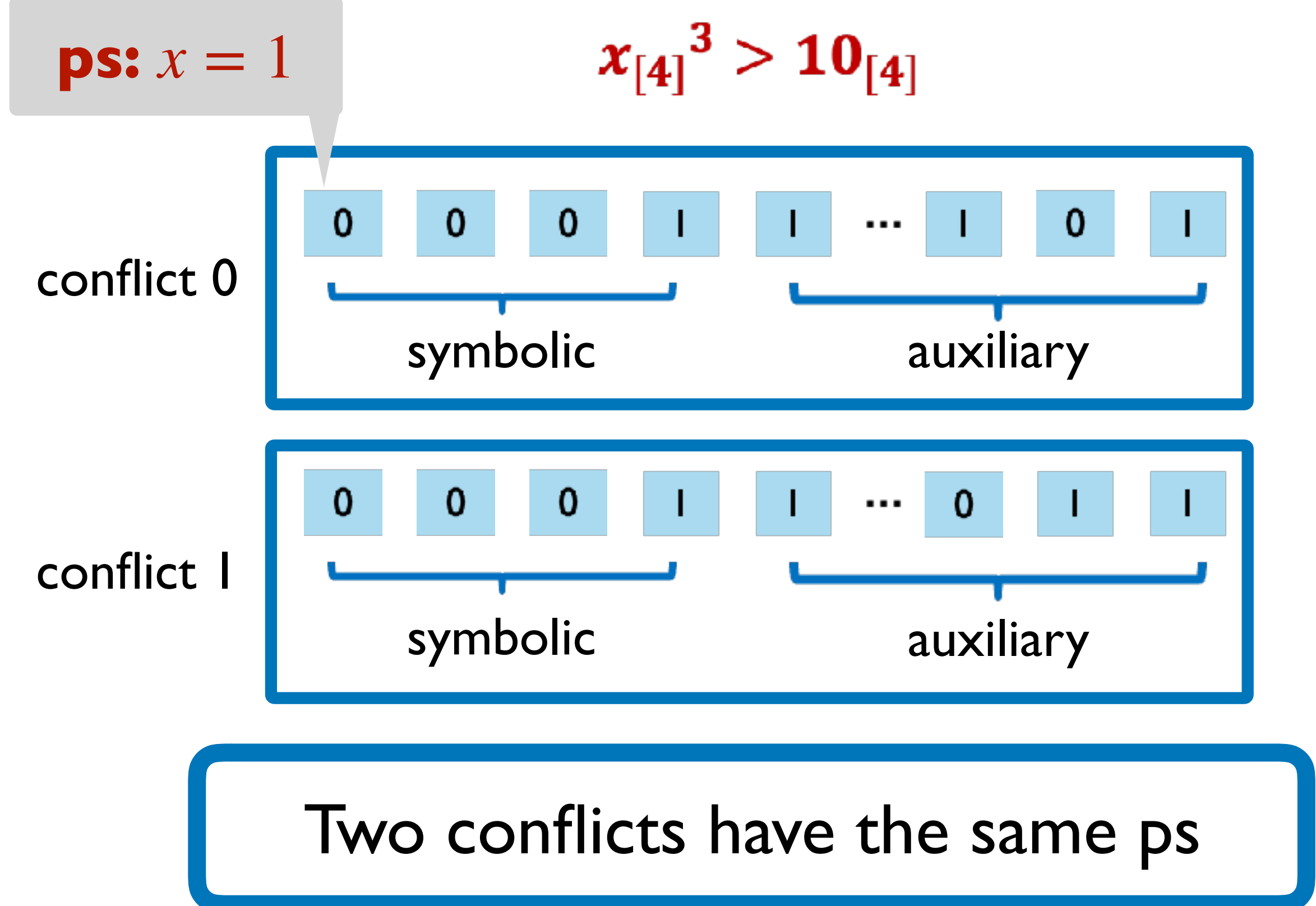
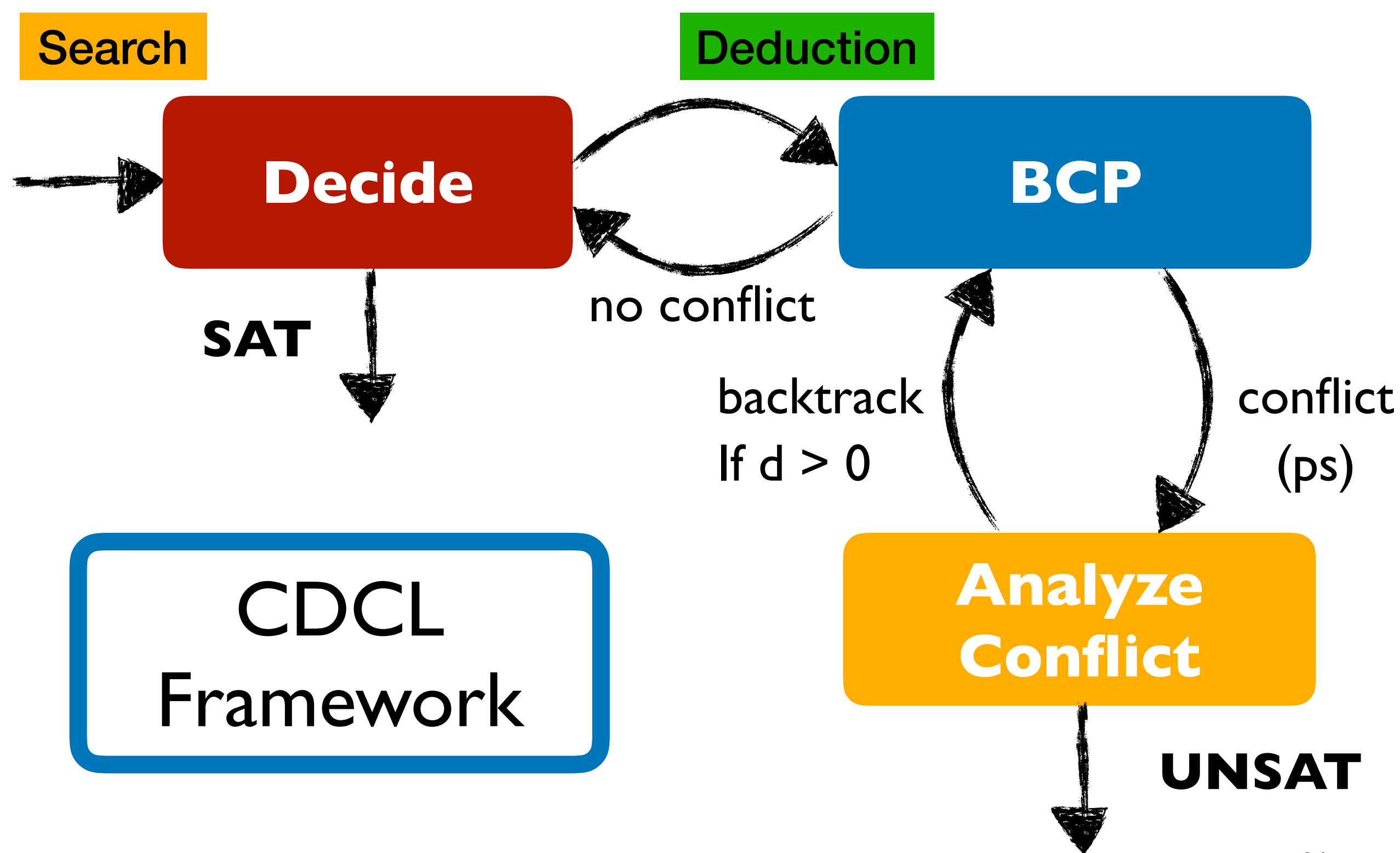
# Internals: Build PS

- Build PS based on **intermediate assignments causing conflicts**



# Internals: Build PS

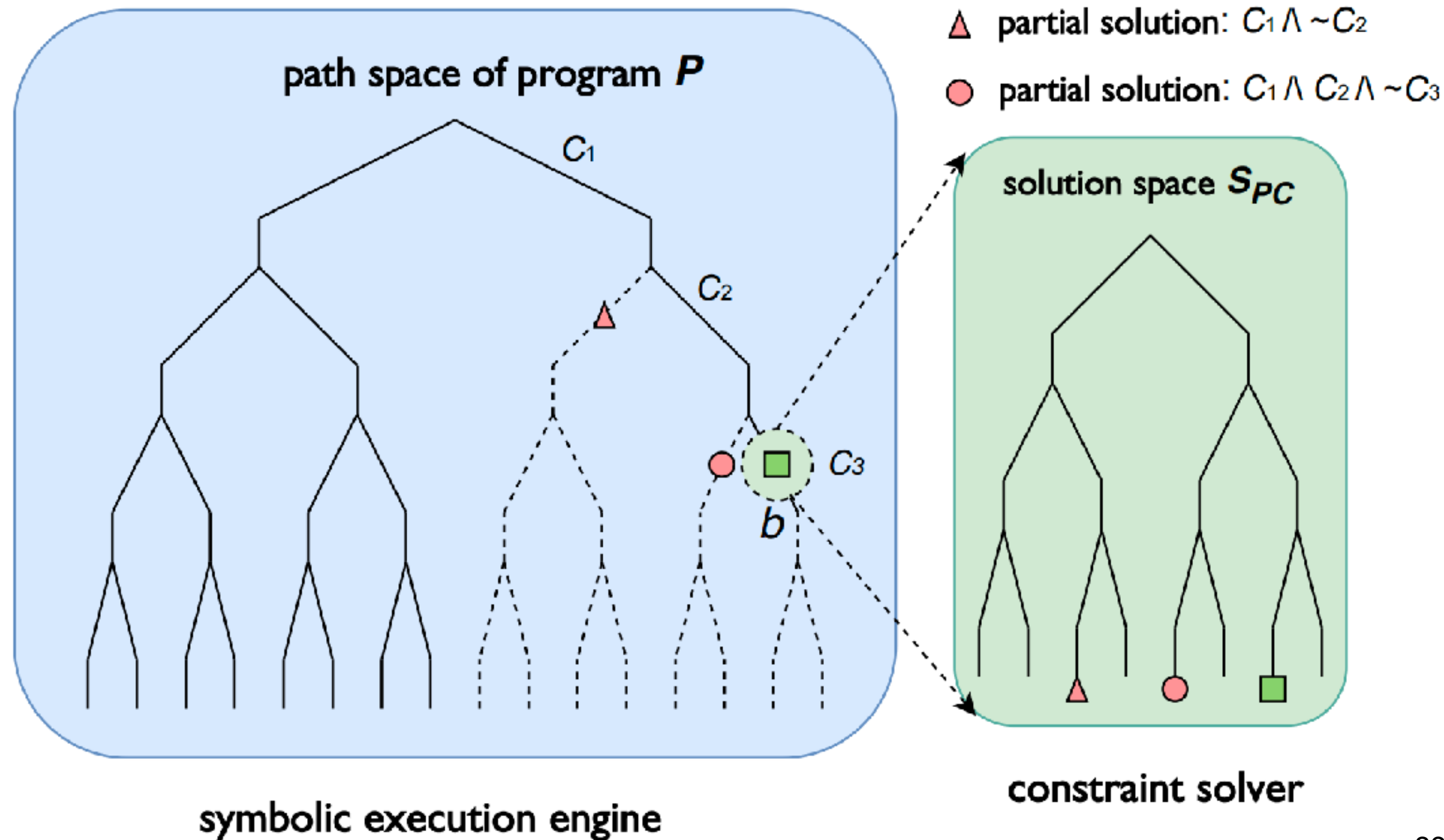
- Build PS based on **intermediate assignments causing conflicts**
- Only reserve PS whose symbolic booleans are different



# Internals: Expand the Cache

- Construct cache entries:  $\text{Prefix}(\varphi) \cup \text{OffThePath}(\varphi)$

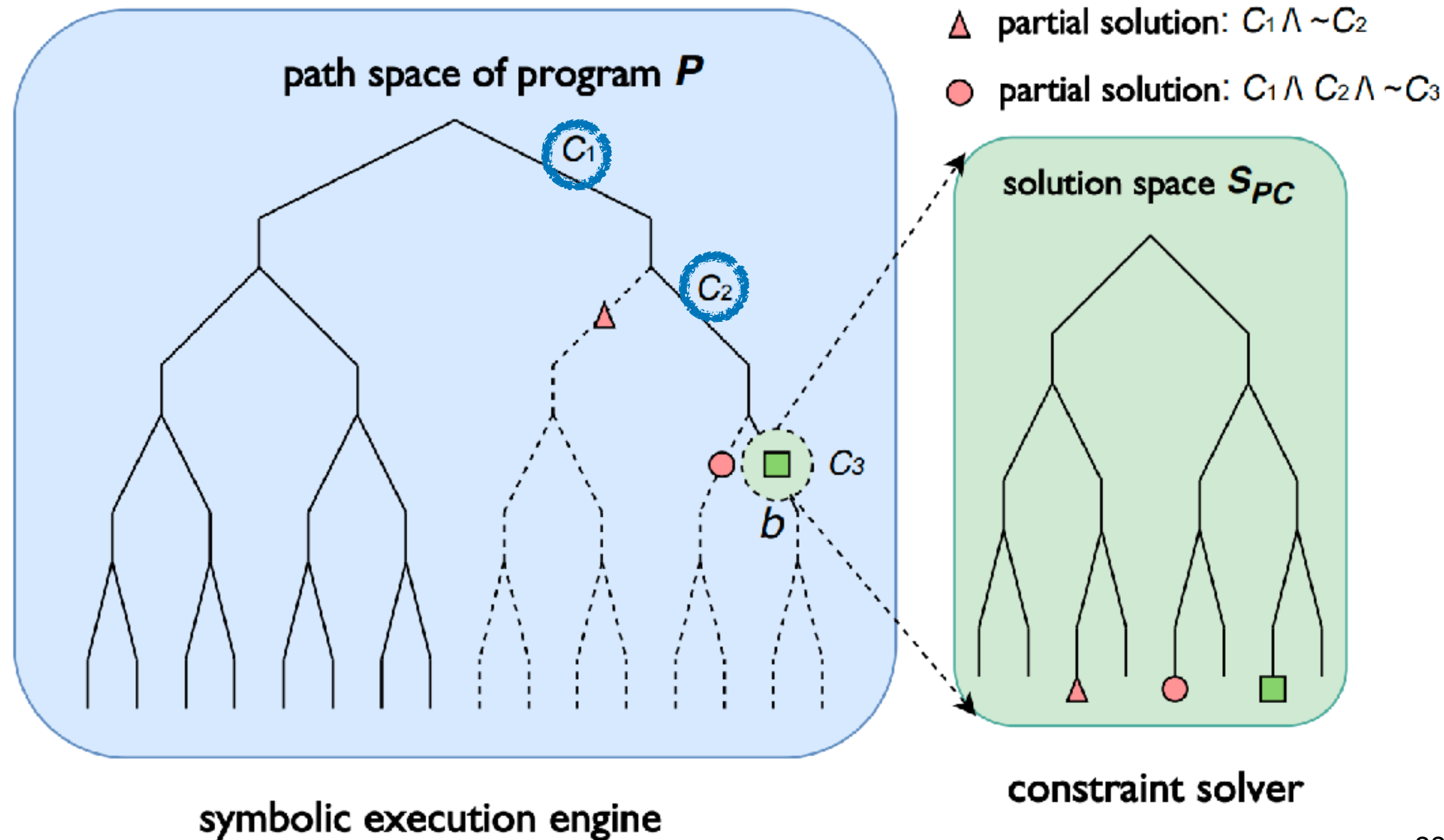
$$\varphi = C_1 \wedge C_2 \wedge C_3$$



# Internals: Expand the Cache

- Construct cache entries:  $\text{Prefix}(\varphi) \cup \text{OffThePath}(\varphi)$

$$\varphi = C_1 \wedge C_2 \wedge C_3$$



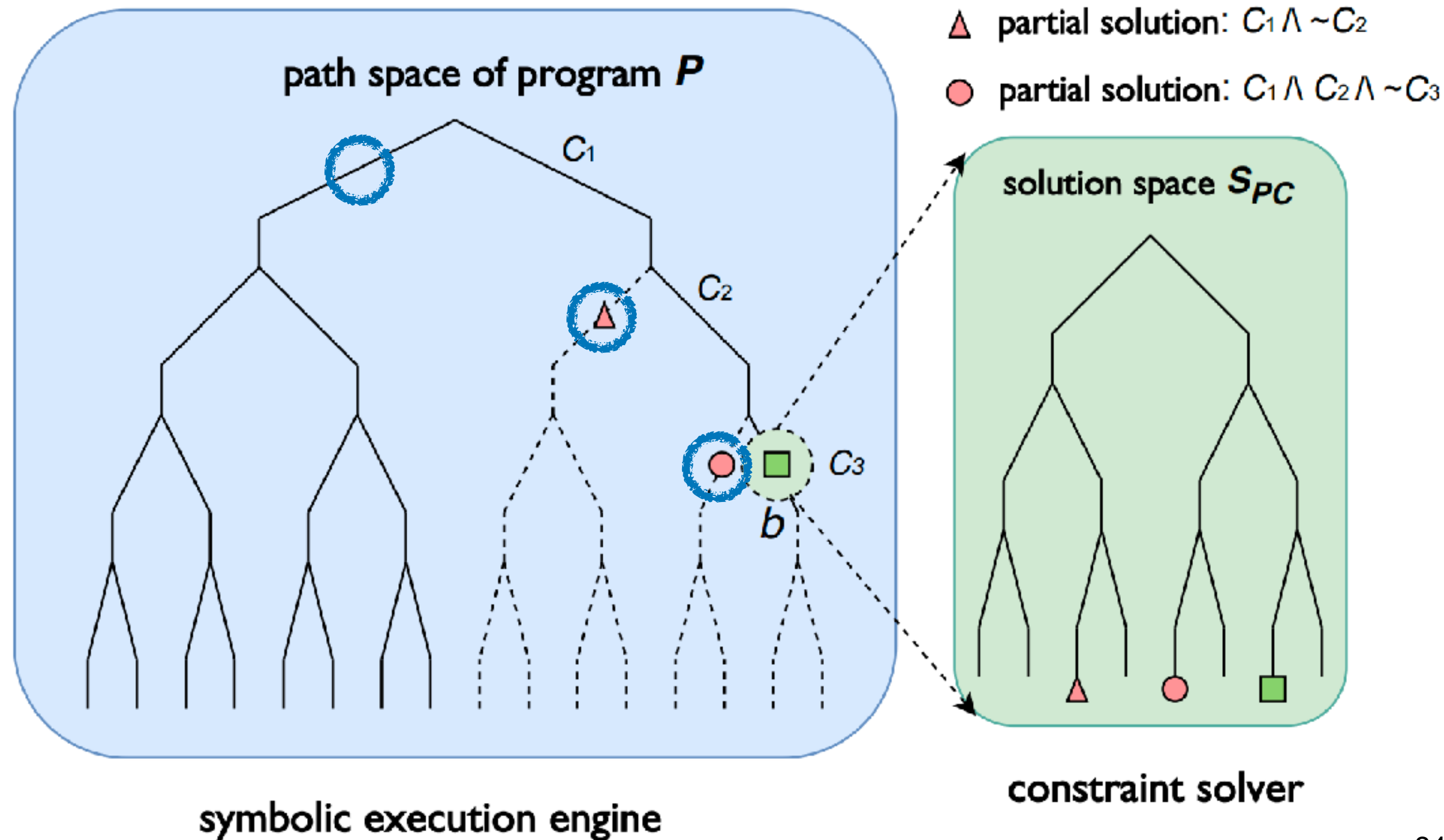
**Prefix( $\varphi$ ):** all prefixes of  $\varphi$

$$\text{Prefix}(\varphi) = \{C_1, C_1 \wedge C_2\}$$

# Internals: Expand the Cache

- Construct cache entries:  $\text{Prefix}(\varphi) \cup \text{OffThePath}(\varphi)$

$$\varphi = C_1 \wedge C_2 \wedge C_3$$



$\text{Prefix}(\varphi)$ : all prefixes of  $\varphi$

$\text{OffThePath}(\varphi)$ : path constraints of  $\varphi$ 's off-the-path branches

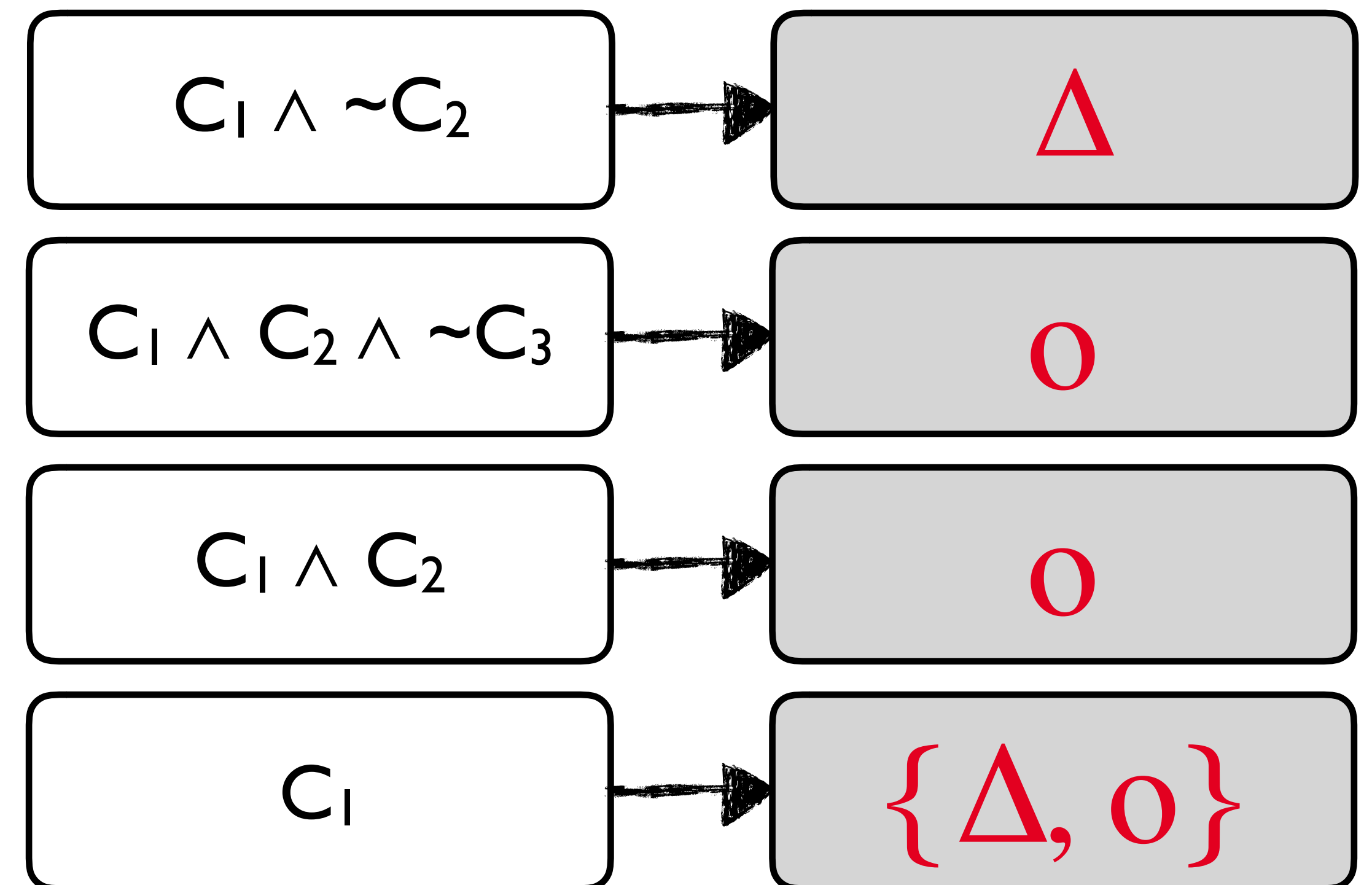
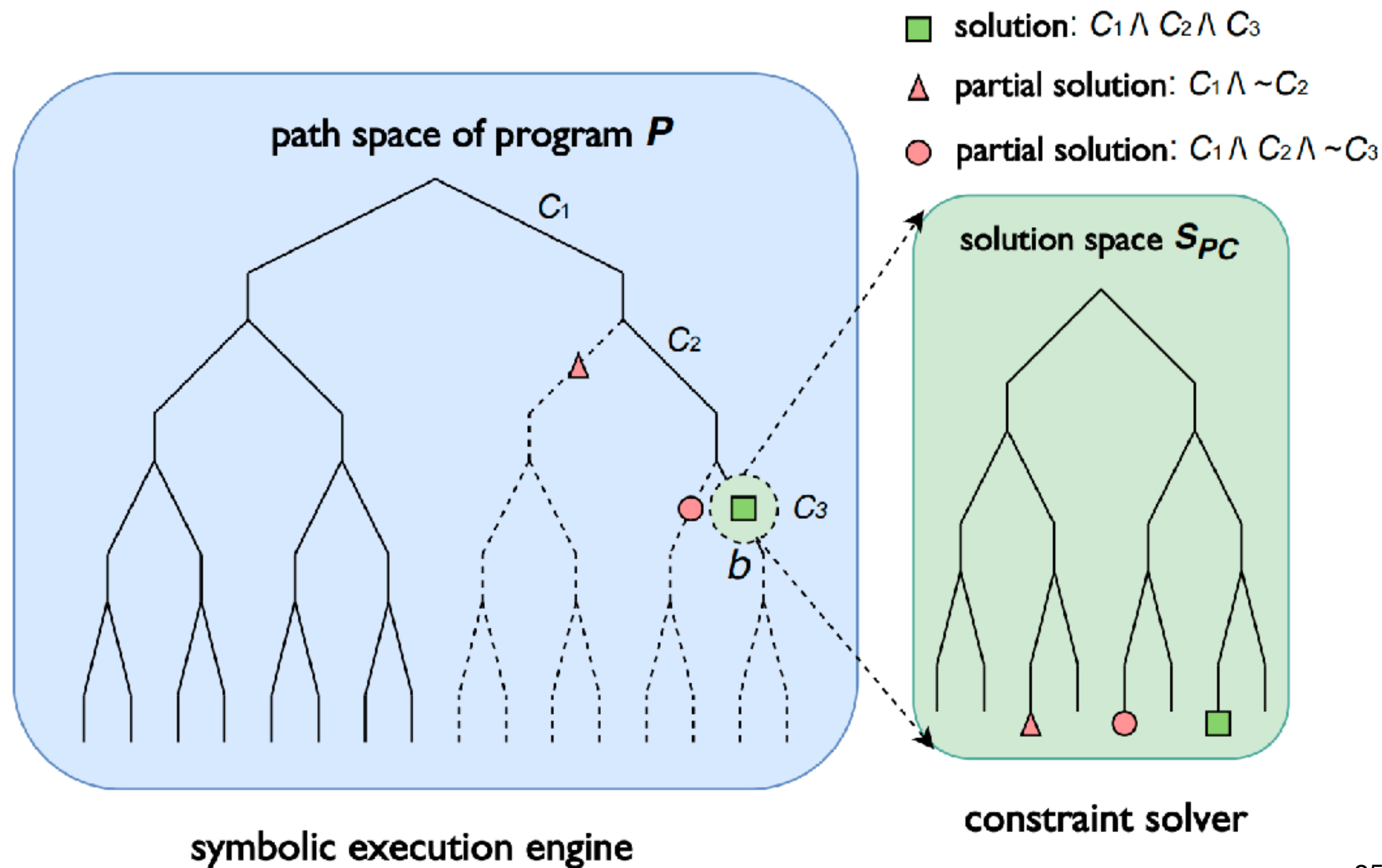
$$\text{Prefix}(\varphi) = \{C_1, C_1 \wedge C_2\}$$

$$\text{OffThePath}(\varphi) = \{\sim C_1, C_1 \wedge \sim C_2, C_1 \wedge C_2 \wedge \sim C_3\}$$



# Internals: Expand the Cache

- Construct cache entries:  $\text{Prefix}(\varphi) \cup \text{OffThePath}(\varphi)$
- Extend the single solution of cache entry to a **solution set**

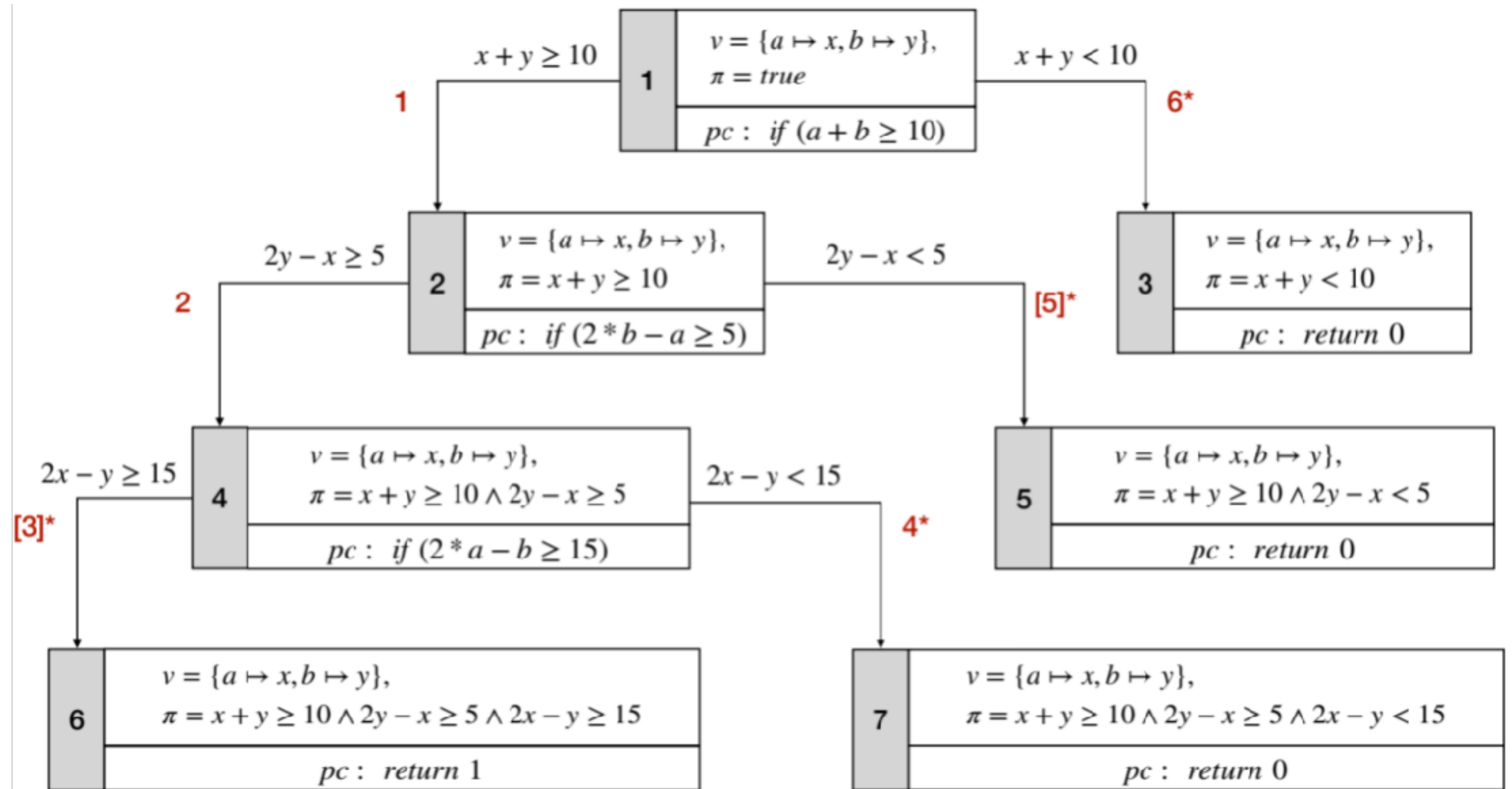


# Toy Example

```

int test(int8_t a, int8_t b) {
  if (a + b >= 10) {
    if (2*b - a >= 5) {
      if (2*a - b >= 15) {
        return 1;
      }
    }
  }
  return 0;
}

```



Depth-First Search (DFS) Heuristic

# Example: Traditional SE

- SE needs to decide the satisfiability of 6 SMT queries
  - $x+y \geq 10$  with solution  $\{x = 10, y = 0\}$
  - $x+y \geq 10 \wedge 2y-x \geq 5$  with solution  $\{x = 26, y = 17\}$
  - $x+y \geq 10 \wedge 2y-x \geq 5 \wedge 2x-y \geq 15$  with solution  $\{x = 20, y = 20\}$
  - $x+y \geq 10 \wedge 2y-x \geq 5 \wedge 2x-y < 15$  with solution  $\{x = 7, y = 31\}$
  - $x+y \geq 10 \wedge 2y-x < 5$  with solution  $\{x = 10, y = 1\}$
  - $x+y < 10$  with solution  $\{x = 0, y = 9\}$

Need 6 times of solving

# Example: Subset-based Caching

- SE needs to decide the satisfiability of 6 SMT queries
  - $x+y \geq 10$  with solution  $\{x = 10, y = 0\}$
  - $x+y \geq 10 \wedge 2y-x \geq 5$  with solution  $\{x = 26, y = 17\}$
  - $x+y \geq 10 \wedge 2y-x \geq 5 \wedge 2x-y \geq 15$  cached by solution  $\{x = 26, y = 17\}$
  - $x+y \geq 10 \wedge 2y-x \geq 5 \wedge 2x-y < 15$  with solution  $\{x = 7, y = 31\}$
  - $x+y \geq 10 \wedge 2y-x < 5$  cached by solution  $\{x = 10, y = 0\}$
  - $x+y < 10$  with solution  $\{x = 0, y = 9\}$

Need 4 times of solving

# Example: PS-Based Caching

- SE needs to decide the satisfiability of 6 SMT queries

- $x + y \geq 10$  with solution  $\{x = 10, y = 0\}$

Too simple to produce any ps

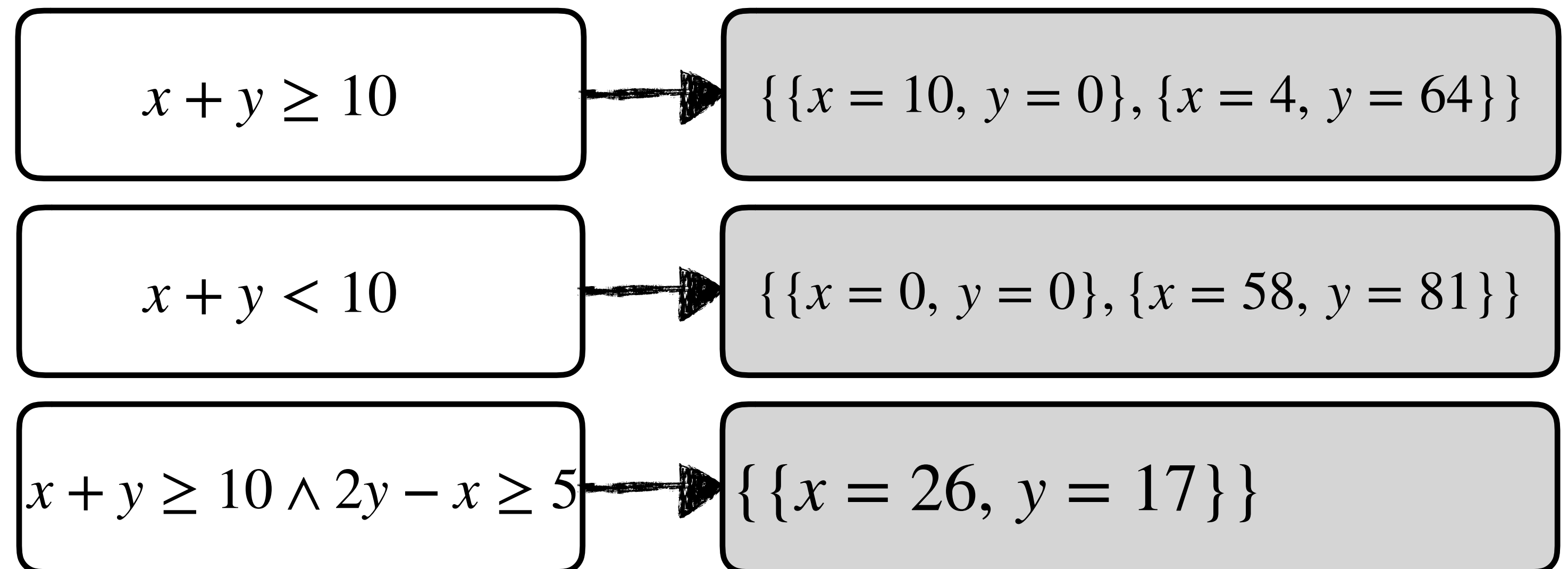
- $x + y \geq 10 \wedge 2y - x \geq 5$  with solution  $\{x = 26, y = 17\}$

Three partial solutions:

$$\{x = 58, y = 81\},$$

$$\{x = 4, y = 64\},$$

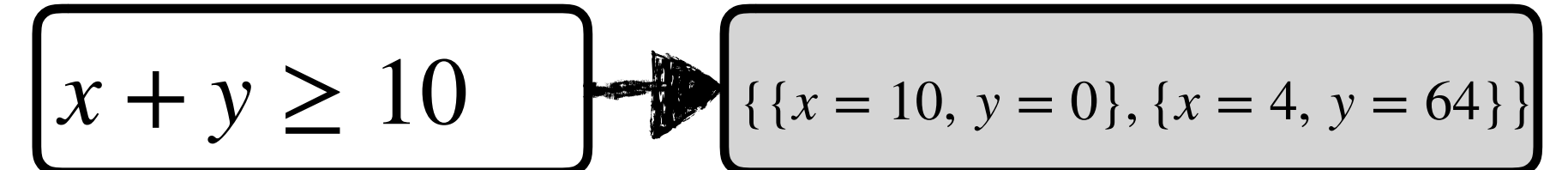
$$\{x = 0, y = 0\}$$



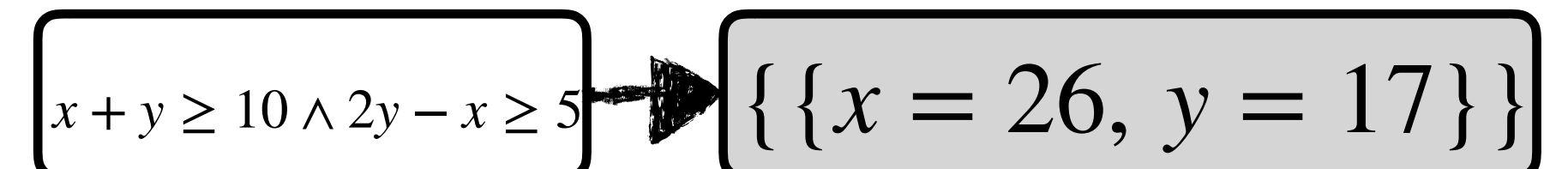
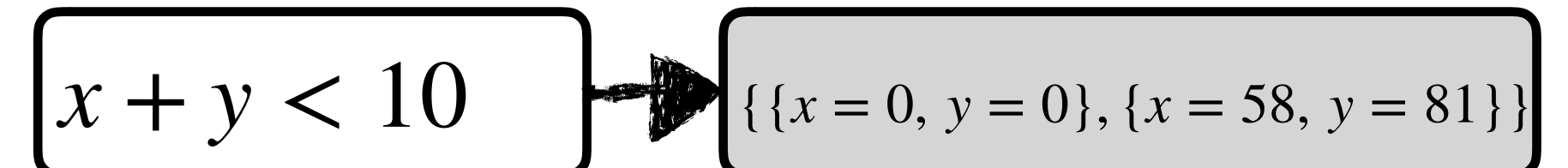
# Example: PS-Based Caching

- SE needs to decide the satisfiability of **6** SMT queries

- $x + y \geq 10$  with solution  $\{x = 10, y = 0\}$



- $x + y \geq 10 \wedge 2y - x \geq 5$  with solution  $\{x = 26, y = 17\}$



- $x + y \geq 10 \wedge 2y - x \geq 5 \wedge 2x - y \geq 15$  **cached by solution  $\{x = 26, y = 17\}$**

- $x + y \geq 10 \wedge 2y - x \geq 5 \wedge 2x - y < 15$  **cached by solution  $\{x = 4, y = 64\}$**

- $x + y \geq 10 \wedge 2y - x < 5$  **cached by solution  $\{x = 10, y = 0\}$**

- $x + y < 10$  **cached by solution  $\{x = 0, y = 0\}$**

**Need 2 times of solving**

# Implementation

- Two types of symbolic executors
  - KLEE / KLEE-Float (**C**): Subset&Supset-based caching
  - Grulia service in SPF & Green (**Java**): Utopia[TSE'21]
- Two SMT solvers
  - STP: external SAT solver (Minisat[SAT'03])
  - Z3: self-customized SAT solver

# Implementation

effectiveness vs. time & memory overhead

- Implement two parameters to achieve **trade-off**
  - $K_p$ : limits the number of ps returned by the solver
  - $K_s$ : limits the maximum size of the solution set



# Evaluation

- Research questions
  - RQ1: Effectiveness - the number of explored paths or states
  - RQ2: Efficiency - path exploration trend
  - RQ3: The impact of parameter tuning ( $K_p$  and  $K_s$ )

# Experimental Setup

<b>Experiments</b>	<b>Symbolic Executor / Timeout</b>	<b>Constraint Solver / Timeout</b>	<b>Kp + Ks</b>	<b>Search Heuristic</b>
QF_ABV based analysis	KLEE / 30min	STP & Minisat / 30s	250 + 50	Depth-First (DFS), Breadth-First (BFS), Random Cover-New (RCN)
QF_ABVFP based analysis	KLEE-Float / 30min	Z3 / 200s	2500 + 2500	
QF_BV based analysis	SPF & Green / 30min	Z3 / 5s	100 + 100	Depth-First (DFS), Breadth-First (BFS)

# Benchmark

- Benchmark
  - QF\_ABV: **15** real-world open-source C programs
  - QF\_ABVFP: **32** randomly chosen GSL functions
  - QF\_BV: **13** Java programs from Green benchmark

# Results of Effectiveness (QF\_ABV)

Programs	Mode	DFS			BFS			RCN		
		#Paths	CHR	TO(s)	#Paths	CHR	TO(s)	#Paths	CHR	TO(s)
apr	P	<b>107(33.8%)</b>	0.847	1.24	84(10.5%)	0.703	0.77	87(29.9%)	0.765	0.87
	O	80	0.793	0.0	76	0.671	0.0	67	0.721	0.0
cmark	P	1790(-7.3%)	0.485	213.49	1569(3.8%)	0.528	28.84	<b>2569(5.9%)</b>	0.576	28.01
	O	1931	0.463	0.0	1511	0.502	0.0	2425	0.546	0.0
fribidi	P	336(23.1%)	0.924	9.75	<b>8427(24.3%)</b>	0.975	6.09	6925(2.2%)	0.971	4.19
	O	273	0.909	0.0	6782	0.966	0.0	6773	0.966	0.0
gas	P	312(2.3%)	0.888	6.03	<b>1541(12.2%)</b>	0.895	1.09	2563(-13.3%)	0.906	0.9
	O	305	0.876	0.0	1373	0.87	0.0	2955	0.906	0.0
json-c	P	409(3.0%)	0.565	65.39	569(28.7%)	0.557	71.71	<b>492(41.8%)</b>	0.556	67.1
	O	397	0.35	0.0	442	0.236	0.0	347	0.209	0.0
libinjection	P	1007(-15.7%)	0.936	1.87	<b>27373(1.7%)</b>	0.936	4.3	16088(1.5%)	0.928	2.77
	O	1194	0.946	0.0	26918	0.939	0.0	15856	0.924	0.0
libtommath	P	38006(5.1%)	0.933	24.62	27214(33.7%)	0.923	8.36	<b>42238(98.7%)</b>	0.96	6.63
	O	36151	0.915	0.0	20347	0.917	0.0	21260	0.964	0.0
m4	P	14938(-26.4%)	0.965	124.29	75509(1.2%)	0.996	131.61	<b>44140(31.4%)</b>	0.99	129.03
	O	20304	0.969	0.0	74645	0.995	0.0	33586	0.983	0.0
discount	P	<b>86474(24.0%)</b>	0.974	25.74	222984(9.8%)	0.999	3.68	274828(4.3%)	0.998	2.62
	O	69764	0.972	0.0	203125	0.998	0.0	263373	0.997	0.0
pacparser	P	36(2.9%)	0.663	12.02	3668(5.9%)	0.96	5.13	<b>5655(9.9%)</b>	0.971	6.95
	O	35	0.641	0.0	3465	0.95	0.0	5145	0.96	0.0
ptx	P	<b>3312(28.4%)</b>	0.985	10.67	151(-5.0%)	0.696	63.74	446(-6.3%)	0.778	29.74
	O	2579	0.982	0.0	159	0.682	0.0	476	0.779	0.0
sha1-cd	P	80(9.6%)	0.596	38.26	<b>1772(68.0%)</b>	0.591	266.96	3180(34.3%)	0.568	551.15
	O	73	0.52	0.0	1055	0.525	0.0	2367	0.52	0.0
smaz	P	52(26.8%)	0.622	34.65	154(51.0%)	0.628	17.81	<b>106(89.3%)</b>	0.619	19.84
	O	41	0.601	0.0	102	0.564	0.0	56	0.58	0.0
sqlite3	P	42(-10.6%)	0.85	8.68	225(-6.2%)	0.823	20.96	<b>173(8.8%)</b>	0.685	11.95
	O	47	0.839	0.0	240	0.801	0.0	159	0.581	0.0
sundown	P	1524(6.6%)	0.792	74.05	16244(0.0%)	0.84	50.07	<b>26906(7.9%)</b>	0.883	46.97
	O	1430	0.723	0.0	16241	0.831	0.0	24929	0.87	0.0

Search Heuristic	Improvement (%)
<b>DFS</b>	7.0
<b>BFS</b>	16.0
<b>RCN</b>	23.1

Improve the numbers of paths for 9 programs under all search heuristics

# Results of Effectiveness (QF\_ABVFP)

GSL Functions	Mode	DFS			BFS			RCN		
		#Paths	CHR	TO(s)	#Paths	CHR	TO(s)	#Paths	CHR	TO(s)
gsl_cdf_beta_P	P	19(0.0%)	0.506	5.2	44(37.5%)	0.617	3.4	53(21.2%)	0.605	5.9
	O	19	0.455	0.0	32	0.465	0.0	52	0.466	0.0
gsl_cdf_beta_Pinv	P	24(0.0%)	0.566	3.2	40(0.0%)	0.661	2.4	64(30.6%)	0.669	4.7
	O	24	0.458	0.0	40	0.503	0.0	49	0.470	0.0
gsl_cdf_cauchy_Pinv	P	17(21.4%)	0.565	3.9	44(10.0%)	0.63	5.0	47(27.0%)	0.663	6.1
	O	14	0.517	0.0	40	0.469	0.0	37	0.466	0.0
gsl_cdf_chiSq_P	P	15(7.1%)	0.581	1.8	39(25.8%)	0.661	3.8	43(22.9%)	0.643	4.1
	O	14	0.478	0.0	31	0.5	0.0	35	0.49	0.0
gsl_cdf_gumbel1_Q	P	14(16.7%)	0.571	1.1	27(42.1%)	0.634	1.3	33(17.9%)	0.602	1.8
	O	12	0.462	0.0	19	0.573	0.0	28	0.538	0.0
gsl_cdf_gumbel1_Qinv	P	16(23.1%)	0.552	1.2	30(11.1%)	0.604	0.7	40(11.1%)	0.614	1.7
	O	13	0.471	0.0	27	0.474	0.0	36	0.476	0.0
gsl_cdf_gumbel2_Pinv	P	24(26.3%)	0.588	0.8	45(7.1%)	0.651	1.5	64(15.3%)	0.675	1.9
	O	19	0.519	0.0	42	0.558	0.0	44	0.507	0.0
gsl_cdf_weibull_Qinv	P	23(9.5%)	0.608	1.1	40(0.0%)	0.629	1.9	44(10.0%)	0.639	1.4
	O	21	0.522	0.0	40	0.562	0.0	40	0.552	0.0
gsl_complex_exp	P	27(58.8%)	0.604	1.7	238(11.7%)	0.841	2.5	510(13.1%)	0.895	3.0
	O	17	0.516	0.0	213	0.794	0.0	274	0.848	0.0
gsl_complex_log	P	27(58.8%)	0.603	1.5	238(11.7%)	0.845	2.6	295(4.6%)	0.892	3.1
	O	17	0.513	0.0	213	0.794	0.0	282	0.85	0.0
gsl_complex_sinh	P	29(45.0%)	0.604	1.5	143(5.1%)	0.721	5.4	284(68.0%)	0.759	8.2
	O	20	0.516	0.0	136	0.681	0.0	169	0.633	0.0
gsl_deriv_forward	P	11(22.2%)	0.589	1.6	13(30.0%)	0.625	1.9	15(25.0%)	0.647	1.6
	O	9	0.462	0.0	10	0.459	0.0	12	0.457	0.0
gsl_diff_forward	P	10(11.1%)	0.541	0.2	10(42.9%)	0.57	0.7	15(50.0%)	0.604	0.6
	O	9	0.513	0.0	7	0.506	0.0	10	0.493	0.0
gsl_eigen_genV_sort	P	17(30.5%)	0.617	1.4	75(27.1%)	0.833	2.9	50(56.2%)	0.656	2.5
	O	13	0.459	0.0	59	0.793	0.0	32	0.519	0.0
gsl_integration_glfixed	P	11(10.0%)	0.569	3.6	22(15.8%)	0.628	3.2	29(20.8%)	0.582	4.3
	O	10	0.477	0.0	19	0.493	0.0	24	0.5	0.0
gsl_integration_qawc	P	138(137.9%)	0.776	195.2	24(0.0%)	0.614	1.0	243(170.0%)	0.796	13.1
	O	58	0.489	0.0	24	0.557	0.0	90	0.628	0.0
gsl_integration_qng	P	12(9.1%)	0.579	3.2	70(311.8%)	0.679	2.8	109(22.5%)	0.739	6.8
	O	11	0.469	0.0	17	0.509	0.0	89	0.619	0.0
gsl_linalg_PTLQ_decomp	P	44(18.9%)	0.538	7.0	113(41.2%)	0.553	2.1	34(13.3%)	0.508	2.1
	O	37	0.494	0.0	80	0.52	0.0	30	0.444	0.0
gsl_linalg_complex_LU_Lndet	P	30(57.9%)	0.726	6.7	79(14.5%)	0.731	2.3	47(-2.1%)	0.627	2.8
	O	19	0.476	0.0	69	0.543	0.0	48	0.491	0.0
gsl_poly_complex_solve_cubic	P	23(1050.0%)	0.776	3.0	25(1150.0%)	0.74	2.1	28(1300.0%)	0.753	2.5
	O	2	0.286	0.0	2	0.286	0.0	2	0.286	0.0
gsl_poly_complex_solve_quadratic	P	20(66.7%)	0.746	4.5	60(160.9%)	0.676	5.8	58(123.1%)	0.697	5.6
	O	12	0.459	0.0	23	0.482	0.0	26	0.484	0.0
gsl_poly_solve_quadratic	P	19(46.2%)	0.719	5.1	26(62.5%)	0.65	2.7	37(50.0%)	0.64	2.5
	O	13	0.463	0.0	16	0.475	0.0	18	0.477	0.0
gsl_sf_airy_Ai_deriv_e	P	92(0.0%)	0.623	26.3	13(0.0%)	0.516	0.1	178(85.4%)	0.827	7.5
	O	92	0.514	0.0	13	0.516	0.0	96	0.5	0.0
gsl_sf_bessel_In_scaled_e	P	95(61.0%)	0.717	30.2	54(28.9%)	0.559	4.9	48(11.6%)	0.614	2.3
	O	59	0.481	0.0	45	0.5	0.0	43	0.51	0.0
gsl_sf_bessel_Inu_e	P	12(0.0%)	0.526	1.4	37(60.9%)	0.633	4.0	35(9.4%)	0.616	4.4
	O	12	0.459	0.0	23	0.496	0.0	32	0.497	0.0
gsl_sf_bessel_Inu_scaled_asymm_unif_e	P	27(107.7%)	0.762	5.7	11(10.0%)	0.647	1.1	18(53.6%)	0.697	1.4
	O	13	0.471	0.0	10	0.468	0.0	11	0.465	0.0
gsl_sf_bessel_Inu_scaled_e	P	12(9.1%)	0.538	1.4	36(50.0%)	0.624	4.3	37(27.6%)	0.638	4.6
	O	11	0.457	0.0	24	0.496	0.0	29	0.496	0.0
gsl_sf_bessel_cos_pi4_e	P	25(92.3%)	0.573	1.2	106(3.9%)	0.685	4.0	139(37.5%)	0.744	5.0
	O	13	0.515	0.0	102	0.636	0.0	101	0.582	0.0
gsl_sf_bessel_sin_pi4_e	P	23(64.3%)	0.572	1.2	106(2.9%)	0.687	3.8	139(44.8%)	0.744	5.0
	O	14	0.511	0.0	103	0.634	0.0	96	0.581	0.0
gsl_sf_dbnye_l_e	P	165(217.3%)	0.827	32.5	79(68.1%)	0.78	11.4	144(185.7%)	0.831	28.6
	O	52	0.492	0.0	47	0.492	0.0	21	0.491	0.0
gsl_sf_elljac_e	P	94(-1.1%)	0.605	88.5	71(9.2%)	0.584	3.3	108(16.1%)	0.634	5.6
	O	95	0.497	0.0	65	0.502	0.0	93	0.493	0.0
gsl_sf_gamma_inc_P_e	P	14(-6.7%)	0.49	1.5	49(14.0%)	0.653	4.8	47(20.5%)	0.665	5.6
	O	15	0.468	0.0	43	0.512	0.0	39	0.497	0.0

Search Heuristic	Improvement (%)
DFS	71.0
BFS	70.8
RCN	93.8

Improve the numbers of paths for 29 functions under all search heuristics

# Results of Effectiveness (QF\_ABVFP)

- The ratios of **UNSAT queries** among those reaching the solver vary a lot
- QF\_ABV: DFS (0.7), BFS (0.6), RCN (0.61)
- QF\_ABVFP: DFS (0.4), BFS (0.47), RCN (0.4)

Search Heuristic	Improvement (%)
DFS	71.0
BFS	70.8
RCN	93.8

The PS-based caching mechanism can only handle SAT queries

# Results of Effectiveness (QF\_BV)

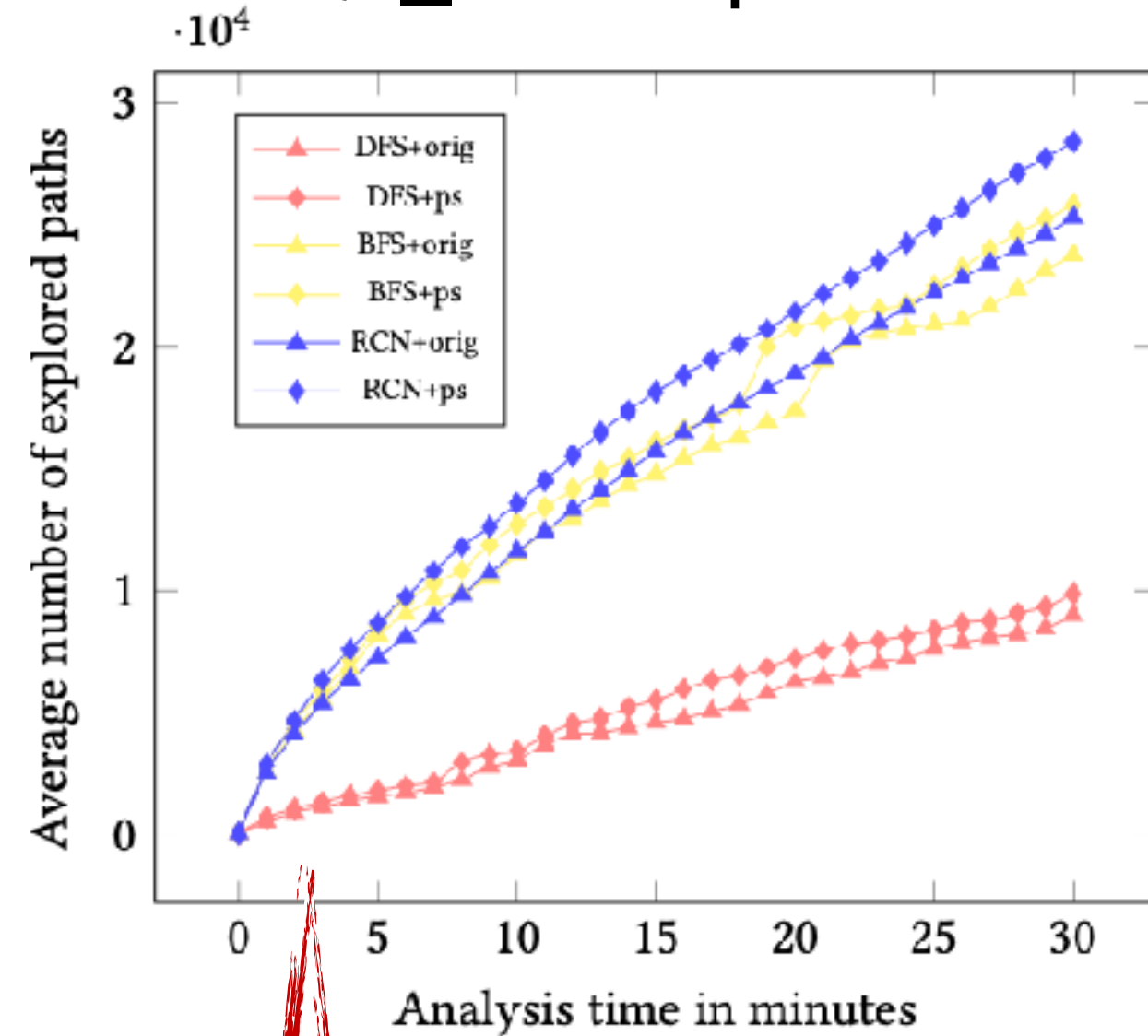
Programs	Mode	DFS				BFS			
		#States	CHR	TO(s)	T(s)	#States	CHR	TO(s)	T(s)
Remainder	P	957(0.0%)	0.761	160.6	<b>285.8</b>	957(0.0%)	0.771	178.6	298.6
	O	957	0.24	0.0	491.0	957	0.156	0.0	551.6
BubbleSort	P	11777(17.9%)	0.283	312.5	1802.0	<b>18255(41.8%)</b>	0.323	159.7	1800.2
	O	9992	0.044	0.0	1800.0	12876	0.144	0.0	1800.0
Dijkstra	P	8242(60.5%)	0.325	113.1	1800.0	<b>8294(63.5%)</b>	0.347	105.3	1800.0
	O	5135	0.116	0.0	1800.0	5072	0.125	0.0	1800.0
NanoXML	P	<b>48520(227.3%)</b>	0.66	6.7	1800.0	99426(51.6%)	0.536	9.1	1800.0
	O	14822	0.362	0.0	1800.0	65598	0.533	0.0	1800.0
SortedListInt	P	<b>79704(377.2%)</b>	0.846	57.8	1800.0	38491(95.5%)	0.817	11.8	1800.0
	O	16703	0.686	0.0	1800.0	19684	0.695	0.0	1800.0
BinTree	P	<b>15227(47.1%)</b>	0.911	14.8	<b>912.6</b>	15227(21.6%)	0.899	17.2	1204.4
	O	10353	0.829	0.0	1800.0	12522	0.842	0.0	1800.0
Triangle	P	2207(0.0%)	0.502	1.0	<b>7.2</b>	2207(0.0%)	0.474	1.5	19.8
	O	2207	0.354	0.0	90.0	2207	0.303	0.0	111.4
Operations	P	15619(0.0%)	0.52	59.7	<b>256.4</b>	15619(0.0%)	0.465	58.7	262.8
	O	15619	0.188	0.0	544.2	15619	0.222	0.0	591.2
Sorting	P	<b>24219(80.0%)</b>	0.561	127.3	1801.0	18714(25.1%)	0.36	48.6	1800.0
	O	13459	0.256	0.0	1800.0	14959	0.181	0.0	1800.0
MagicIndex	P	<b>7202(52.4%)</b>	0.944	0.2	<b>9.2</b>	7202(0.0%)	0.956	0.2	10.8
	O	4726	0.551	0.0	1788.6	7202	0.786	0.0	871.2
BinomialHeap	P	<b>47461(106.5%)</b>	0.939	86.7	<b>1435.4</b>	47303(21.1%)	0.939	65.6	1493.2
	O	22978	0.885	0.0	1800.0	39063	0.925	0.0	1800.0
TreeMap	P	24640(120.2%)	0.942	58.7	1800.0	<b>61411(131.7%)</b>	0.961	30.1	1800.0
	O	11188	0.888	0.0	1800.0	26499	0.93	0.0	1800.0
Median	P	12452(53.5%)	0.29	313.7	1802.0	<b>17504(59.2%)</b>	0.325	156.9	1800.0
	O	8111	0.046	0.0	1800.0	10994	0.14	0.0	1800.0

Search Heuristic	Improvement (%)
<b>DFS</b>	<b>114.3</b>
<b>BFS</b>	<b>56.8</b>

- The speedups of **early completed tasks**
- DFS: 1.72x ~ 12.5x
- BFS: 1.85x ~ 80.67x

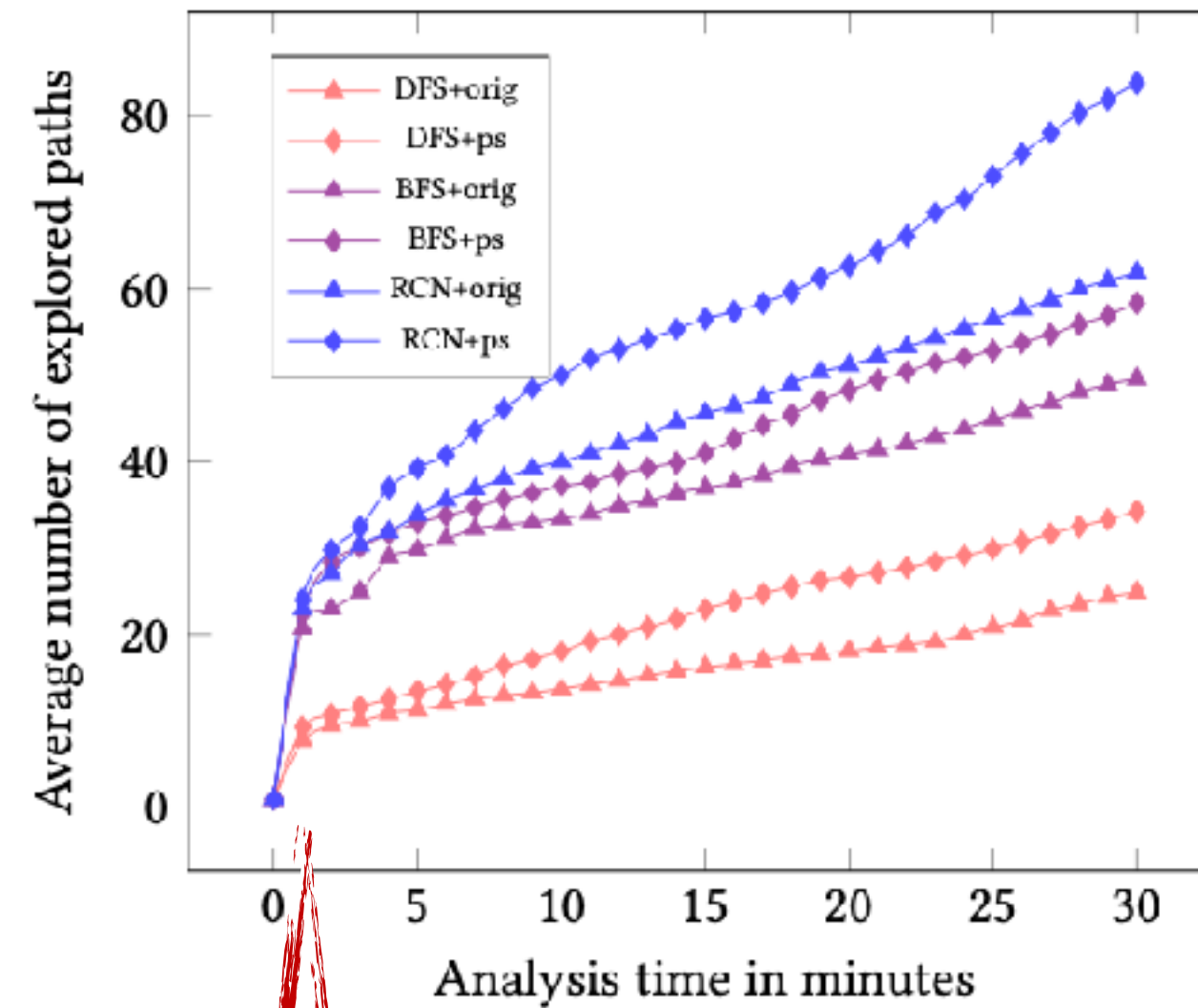
# Results of Efficiency

QF\_ABV Experiment



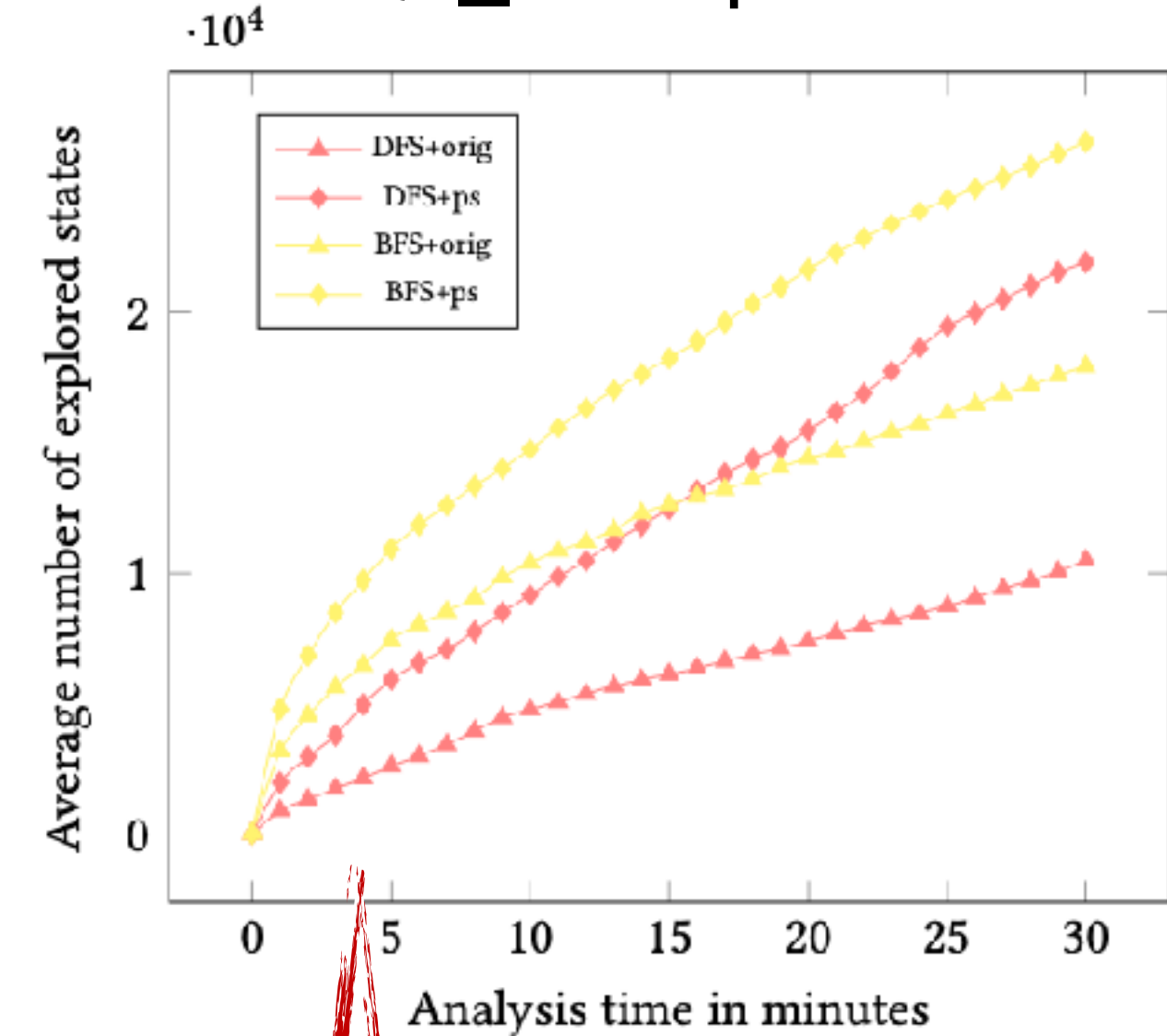
**Speedup**  
**DFS: 1.07x**  
**BFS: 1.11x**  
**RCN: 1.15x**

QF\_ABVFP Experiment



**Speedup**  
**DFS: 1.67x**  
**BFS: 1.36x**  
**RCN: 1.50x**

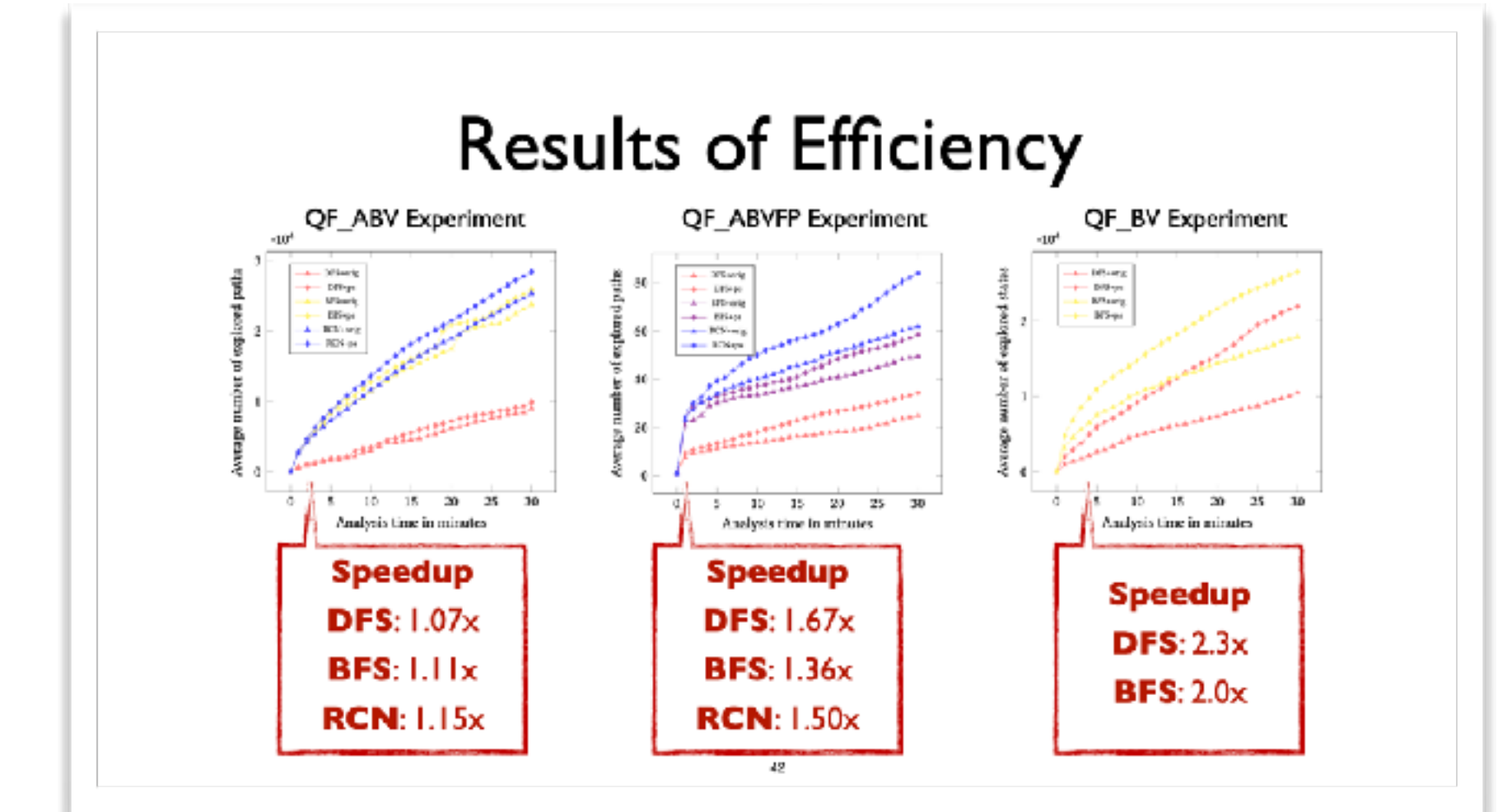
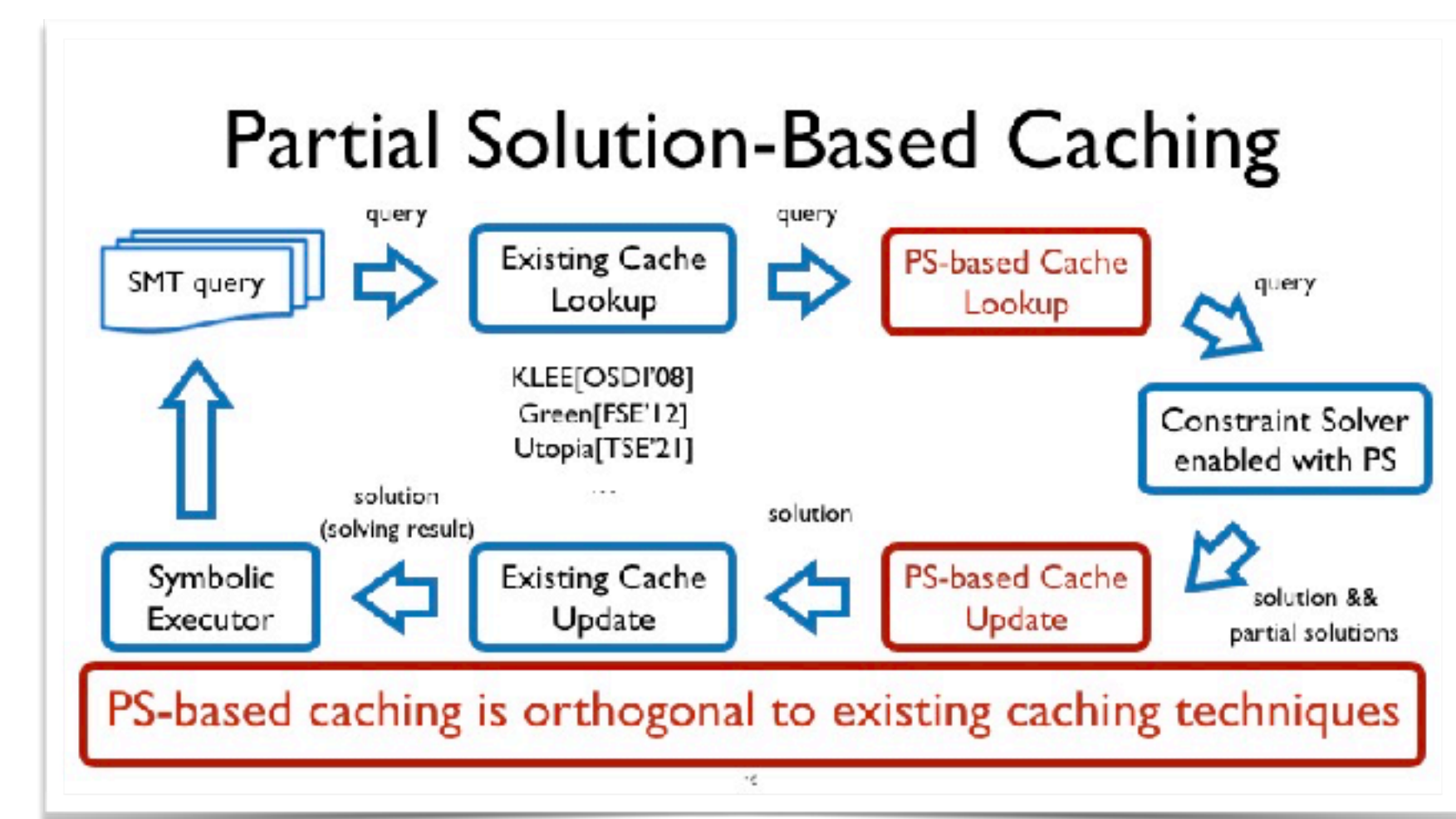
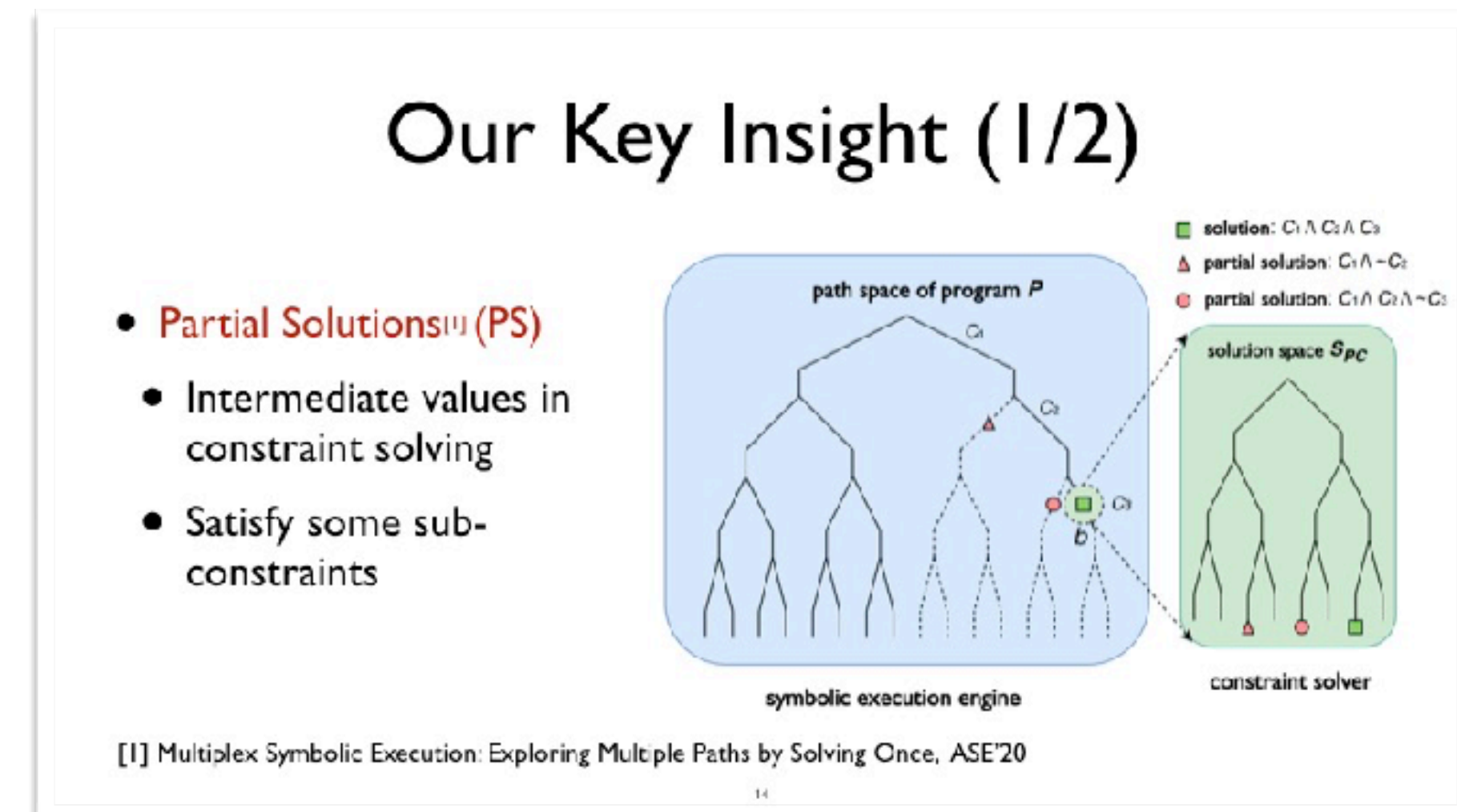
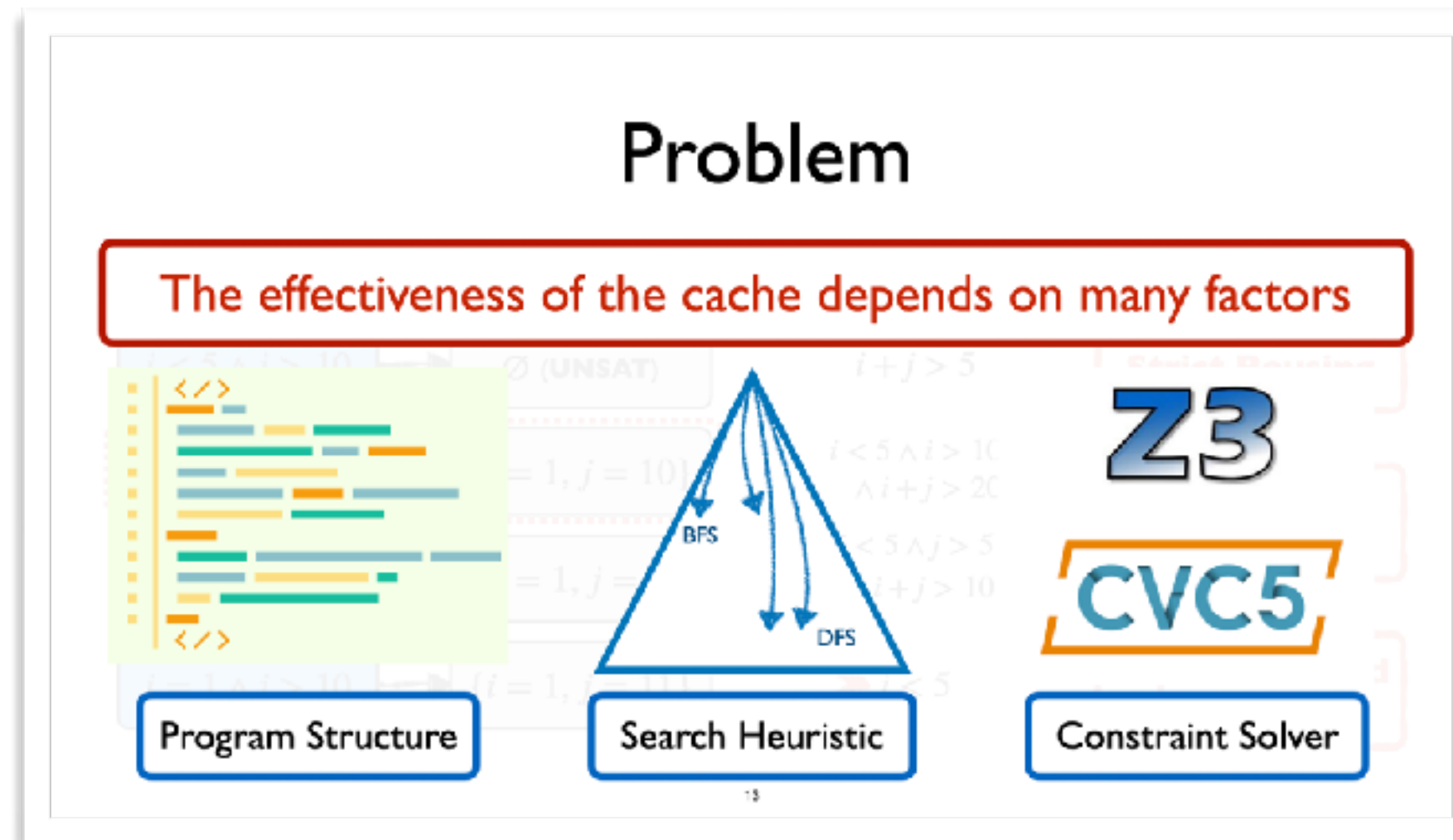
QF\_BV Experiment



**Speedup**  
**DFS: 2.3x**  
**BFS: 2.0x**



# Conclusion



Thank you!  
Q&A

**Artifact**



<https://github.com/zbchen/pscache>